1 Units, Dimensions and Error Analysis

1.1 Units and Dimensions

Chapter Snapshot

- Units and Dimensions
- Significant Figures
- Error Analysis

To measure a physical quantity we need some standard unit of that quantity. The measurement of the quantity is mentioned in two parts, the first part gives how many times of the standard unit and the second part gives the name of the unit. Thus, suppose I say that length of this wire is 5 m. The numeric part 5 says that it is 5 times of the unit of length and the second part metre says that unit chosen here is metre.

Fundamental and Derived Quantities

There are a large number of physical quantities and every quantity needs a unit.

However, not all the quantities are independent. For example, if a unit of length is defined, a unit of volume is automatically obtained. Thus, we can define a set of fundamental quantities and all other quantities may be expressed in terms of the fundamental quantities. Fundamental quantities are only seven in numbers. Unit of all other quantities can be expressed in terms of the units of these seven quantities by multiplication or division.

Many different choices can be made for the fundamental quantities. For example, if we take length and time as the fundamental quantities, then speed is a derived quantity and if we take speed and time as fundamental quantities then length is a derived quantity.

Several system of units are in use over the world. The units defined for the fundamental quantities are called fundamental units and those obtained for derived quantities are called the derived units.

SI Units

In 1971, General Conference on Weight and Measures held its meeting and decided a system of units which is known as the International System of Units. It is abbreviated as SI from the French name Le Systems International *d'* Unites. This system is widely used throughout the world. Table below gives the seven fundamental quantities and their SI units.

Table 1.1

S. No.	Quantity	SI Unit	Symbol
1.	Length	metre	m
2.	Mass	kilogram	kg
3.	Time	second	s
4.	Electric current	ampere	A
5.	Thermodynamic temperature	kelvin	K
6.	Amount of substance	mole	mol
7.	Luminous intensity	candela	cd

Two **supplementary** units namely plane angle and solid angle are also defined. Their units are radian (rad) and steradian (st) respectively.

(i) **CGS System** In this system, the units of length, mass and time are centimetre (cm), gram (g) and second (s) respectively. The unit of force is dyne and that of work or energy is erg.

(ii) **FPS System** In this system, the units of length, mass and time are foot, pound and second. The unit of force in this system is poundal.

SI Prefixes

The most commonly used prefixes are given below in tabular form.

Table 1.2

Power of 10	Prefix	Symbol
6	mega	M
3	kilo	k
– 2	centi	c
– 3	mili	m
– 6	micro	μ
– 9	nano	n
– 12	pico	p

Definitions of Some Important SI Units

(i) **Metre** 1 m = 1,650,763.73 wavelengths in vacuum, of radiation corresponding to orange red light of krypton-86.

(ii) **Second** 1s = 9,192,631,770 time periods of a particular radiation from cesium-133 atom.

(iii) **Kilogram** 1 kg = mass of 1 L volume of water at 4°C.

(iv) **Ampere** It is the current which when flows through two infinitely long straight conductors of negligible cross-section placed at a distance of 1 m in vacuum produces a force of 2×10^{-7} N/m between them.

(v) **Kelvin** 1 K = 1 / 273.16 part of the thermodynamic temperature of triple point of water.

(vi) **Mole** It is the amount of substance of a system which contains as many elementary particles (atoms, molecules, ions etc.) as there are atoms in 12 g of carbon-12.

(vii) **Candela** It is luminous intensity in a perpendicular direction of a surface of $\left(\frac{1}{600000}\right) m^2$ of a black body at the temperature of freezing platinum under a pressure of 1.013×10^5 N/m^2.

(viii) **Radian** It is the plane angle between two radii of a circle which cut-off on the circumference, an arc equal in length to the radius.

(ix) **Steradian** The steradian is the solid angle which having its vertex at the centre of the sphere, cut-off an area of the surface of sphere equal to that of a square with sides of length equal to the radius of the sphere.

Dimensions

Dimensions of a physical quantity are the powers to which the fundamental quantities must be raised to represent the given physical quantity.

For example, $\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{\text{mass}}{(\text{length})^3}$

or $\text{density} = (\text{mass})\ (\text{length})^{-3}$ ...(i)

Thus, the dimensions of density are 1 in mass and –3 in length. The dimensions of all other fundamental quantities are zero.

For convenience, the fundamental quantities are represented by one letter symbols. Generally mass is denoted by M, length by L, time by T and electric current by A.

The thermodynamic temperature, the amount of substance and the luminous intensity are denoted by the symbols of their units K, mol and cd respectively. The physical quantity that is expressed in terms of the base quantities is enclosed in square brackets.

Thus, Eq. (i) can be written as

$$[\text{density}] = [ML^{-3}]$$

Such an expression for a physical quantity in terms of the fundamental quantities is called the dimensional formula.

Here, it is worthnoting that constants such as 5, π or trigonometrical functions such as sin θ, cos θ etc., have no units and dimensions.

$$[\sin\theta] = [\cos\theta] = [\tan\theta] = [\log x] = [e^x] = [M^0L^0T^0]$$

Table 1.3 given below gives the dimensional formulae and SI units of some physical quantities frequently used in physics.

Table 1.3

S. No.	Physical Quantity	SI Units	Dimensional Formula
1.	Velocity = displacement/time	m/s	$[M^0LT^{-1}]$
2.	Acceleration = velocity/time	m/s^2	$[M^0LT^{-2}]$
3.	Force = mass × acceleration	kg-m/s^2 = newton or N	$[MLT^{-2}]$
4.	Work = force × displacement	kg-m^2/s^2 = N-m = joule or J	$[ML^2T^{-2}]$
5.	Energy	J	$[ML^2T^{-2}]$
6.	Torque = force × perpendicular distance	N-m	$[ML^2T^{-2}]$
7.	Power = work/time	J/s or watt	$[ML^2T^{-3}]$
8.	Momentum = mass × velocity	kg-m/s	$[MLT^{-1}]$
9.	Impulse = force × time	N-s	$[MLT^{-1}]$
10.	Angle = arc/radius	radian or rad	$[M^0L^0T^0]$
11.	Strain = $\frac{\Delta L}{L}$ or $\frac{\Delta V}{V}$	No units	$[M^0L^0T^0]$
12.	Stress = force/area	N/m^2	$[ML^{-1}T^{-2}]$
13.	Pressure = force/area	N/m^2	$[ML^{-1}T^{-2}]$

S. No.	Physical Quantity	SI Units	Dimensional Formula
14.	Modulus of elasticity = stress/strain	N/m^2	$[ML^{-1}T^{-2}]$
15.	Frequency = 1/time period	per sec or hertz (Hz)	$[M^0L^0T^1]$
16.	Angular velocity = angle/time	rad/s	$[M^0L^0T^{-1}]$
17.	Moment of inertia = (mass) × (distance)^2	kg-m^2	$[ML^2T^0]$
18.	Surface tension = force/length	N/m	$[ML^0T^{-2}]$
19.	Gravitational constant = $\frac{\text{force} \times (\text{distance})^2}{(\text{mass})^2}$	N-m^2/kg^2	$[M^{-1}L^3T^{-2}]$
20.	Angular momentum	kg-m^2/s	$[ML^2T^{-1}]$
21.	Coefficient of viscosity	N-s/m^2	$[ML^{-1}T^{-1}]$
22.	Planck's constant	J-s	$[ML^2T^{-1}]$
23.	Specific heat (s)	J/kg-K	$[M^0L^2T^{-2}\theta^{-1}]$
24.	Coefficient of thermal conductivity (K)	watt/m-K	$[MLT^{-3}\theta^{-1}]$
25.	Gas constant (R)	J/mol-K	$[ML^2T^{-2}\theta^{-1}\text{mol}^{-1}]$
26.	Boltzmann constant (k)	J/K	$[ML^2T^{-2}\theta^{-1}]$
27.	Wien's constant (b)	m-K	$[L\theta]$
28.	Stefan's constant (σ)	watt/m^2-K^4	$[MLT^{-3}\theta^{-4}]$
29.	Electric charge	C	[AT]
30.	Electric intensity	N/C	$[MLT^{-3}A^{-1}]$
31.	Electric potential	volt (V)	$[ML^2T^{-3}A^{-1}]$
32.	Capacitance	farad (F)	$[M^{-1}L^{-2}T^4A^2]$
33.	Permittivity of free space	$C^2N^{-1}m^{-2}$	$[M^{-1}L^{-3}T^4A^2]$
34.	Electric dipole moment	C-m	[LTA]
35.	Resistance	Ohm	$[ML^2T^{-3}A^{-2}]$
36.	Magnetic field	tesla (T) or weber/m^2 (Wb/m^2)	$[MT^{-2}A^{-1}]$
37.	Coefficient of self-induction	henry (H)	$[ML^2T^{-2}A^{-2}]$

Key-Terms for **Concepts**

- Astronomical unit
  1 AU = mean distance of earth from sun
  $\approx 1.5 \times 10^{11}$ m
- Light year
  1 ly = distance travelled by light in vacuum in 1 year
  $= 9.46 \times 10^{15}$ m
- Parsec
  1 parsec $= 3.07 \times 10^{16}$ m = 3.26 light year
- X-ray unit
  1 U $= 10^{-13}$ m
- 1 shake $= 10^{-8}$ s
- 1 bar $= 10^5$ N/m^2 $= 10^5$ pascal
- 1 torr = 1 mm of Hg = 133.3 Pa
- 1 barn $= 10^{-28}$ m^2
- 1 horse power = 746 W
- 1 pound = 453.6 g = 0.4536 kg

Example 1.1 *Find the dimensional formulae of*

(a) coefficient of viscosity η
(b) charge q
(c) potential V
(d) capacitance C, and
(e) resistance R.

Some of the equations containing these quantities are

$$F = -\eta A\left(\frac{\Delta v}{\Delta l}\right), \quad q = It, \quad U = VIt,$$

$$q = CV \quad \text{and} \quad V = IR$$

where A denotes the area, v the velocity, l is the length, I the electric current, t the time and U the energy.

Solution (a) $\eta = -\frac{F}{A}\frac{\Delta l}{\Delta v}$

$$\therefore \quad [\eta] = \frac{[F][l]}{[A][v]} = \frac{[MLT^{-2}][L]}{[L^2][LT^{-1}]}$$

$$= [ML^{-1}T^{-1}]$$

(b) $q = It$

$$\therefore \quad [q] = [I][t] = [AT]$$

(c) $U = VIt$

$\therefore \quad V = \dfrac{U}{It}$

or $[V] = \dfrac{[U]}{[I][t]} = \dfrac{[ML^2T^{-2}]}{[A][T]} = [ML^2T^{-3}A^{-1}]$

(d) $q = CV$

$\therefore \quad C = \dfrac{q}{V}$

or $[C] = \dfrac{[q]}{[V]} = \dfrac{[AT]}{[ML^2T^{-3}A^{-1}]} = [M^{-2}L^{-2}T^4A^2]$

(e) $V = IR$

$\therefore \quad R = \dfrac{V}{I}$

or $[R] = \dfrac{[V]}{[I]} = \dfrac{[ML^2T^{-3}A^{-1}]}{[A]} = [ML^2T^{-3}A^{-2}]$

Uses of Dimensions

Theory of dimensions have following main uses

(i) **Conversion of units** This is based on the fact that the product of the numerical value (n) and its corresponding unit (u) is a constant, *i.e.*,

$n\,[\mathrm{u}] = \text{constant} \quad \text{or} \quad n_1\,[\mathrm{u}_1] = n_2\,[\mathrm{u}_2]$

Suppose the dimensions of a physical quantity are a in mass, b in length and c in time. If the fundamental units in one system are M_1, L_1 and T_1 and in the other system are M_2, L_2 and T_2 respectively. Then, we can write

$$n_1\,[M_1^a\,L_1^b\,T_1^c] = n_2\,[M_2^a\,L_2^b\,T_2^c] \qquad \ldots\text{(i)}$$

Here, n_1 and n_2 are the numerical values in two system of units respectively. Using Eq. (i), we can convert the numerical value of a physical quantity from one system of units into the other system.

Example 1.2 *The value of gravitational constant is $G = 6.67 \times 10^{-11}$ N-m^2/kg^2 in SI units. Convert it into CGS system of units.*

Solution The dimensional formula of G is $[M^{-1}L^3T^{-2}]$.

Using Eq. (i), *i.e.*,

$$n_1[M_1^{-1}L_1^3T_1^{-2}] = n_2[M_2^{-1}L_2^3T_2^{-2}]$$

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^{-1}\left[\frac{L_1}{L_2}\right]^{3}\left[\frac{T_1}{T_2}\right]^{-2}$$

Here, $n_1 = 6.67 \times 10^{-11}$

$M_1 = 1\text{ kg}, M_2 = 1\text{ g} = 10^{-3}\text{ kg}$

$L_1 = 1\text{ m}, L_2 = 1\text{ cm} = 10^{-2}\text{ m},$

$T_1 = T_2 = 1\text{ s}$

Substituting in the above equation, we get

$$n_2 = 6.67 \times 10^{-11}\left[\frac{1\text{ kg}}{10^{-3}\text{ kg}}\right]^{-1}\left[\frac{1\text{ m}}{10^{-2}\text{ m}}\right]^{3}\left[\frac{1\text{ s}}{1\text{ s}}\right]^{-2}$$

or $n_2 = 6.67 \times 10^{-8}$

Thus, value of G in CGS system of units is 6.67×10^{-8} dyne cm^2/g^2.

(ii) **To check the dimensional correctness of a given physical equation** Every physical equation should be dimensionally balanced. This is called the 'Principle of Homogeneity'. The dimensions of each term on both sides of an equation must be the same. On this basis we can judge whether a given equation is correct or not. But a dimensionally correct equation may or may not be physically correct.

Example 1.3 *Show that the expression of the time period T of a simple pendulum of length l given by $T = 2\pi\sqrt{l/g}$ is dimensionally correct.*

Solution $T = 2\pi\sqrt{\dfrac{l}{g}}$

Dimensionally, $[T] = \sqrt{\dfrac{[L]}{[LT^{-2}]}} = [T]$

As in the above equation, the dimensions of both sides are same. The given formula is dimensionally correct.

Principle of Homogeneity of Dimensions

This principle states that the dimensions of all the terms in a physical expression should be same. For example, in the physical expression $s = ut + \dfrac{1}{2}at^2$, the dimensions of s, ut and $\dfrac{1}{2}at^2$ all are same.

> **Note** The physical quantities separated by the symbols $+, -, =, >, <$ etc, have the same dimensions.

Example 1.4 *The velocity v of a particle depends upon the time t according to the equation $v = a + bt + \dfrac{c}{d+t}$. Write the dimensions of a, b, c and d.*

Solution From principle of homogeneity

$[a] = [v]$

or $[a] = [LT^{-1}]$

$[bt] = [v]$

or $[b] = \dfrac{[v]}{[t]} = \dfrac{[LT^{-1}]}{[T]}$

or $[b] = [LT^{-2}]$

Similarly, $[d] = [t] = [T]$

Further, $\dfrac{[c]}{[d+t]} = [v]$ or $[c] = [v][d+t]$

or $[c] = [LT^{-1}][T]$

or $[c] = [L]$

(iii) **To establish the relation among various physical quantities** If we know the factors on which a given physical quantity may depend, we can find a formula relating the quantity with those factors. Let us take an example.

Example 1.5 *The frequency (f) of a stretched string depends upon the tension F (dimensions of force), length l of the string* and the mass per unit length μ of string. Derive the formula for frequency.

Solution Suppose, that the frequency f depends on the tension raised to the power a, length raised to the power b and mass per unit length raised to the power c.

Then, $$f \propto (F)^a \, (l)^b \, (\mu)^c$$

or $$f = k\,(F)^a \,(l)^b \,(\mu)^c \qquad \ldots\text{(i)}$$

Here, k is a dimensionless constant.

Thus, $$[f] = [F]^a \, [l]^b \, [\mu]^c$$

or $$[M^0L^0T^{-1}] = [MLT^{-2}]^a \, [L]^b \, [ML^{-1}]^c$$

or $$[M^0L^0T^{-1}] = [M^{a+c} L^{a+b-c} T^{-2a}]$$

For dimensional balance, the dimensions on both sides should be same.

Thus, $$a + c = 0 \qquad \ldots\text{(ii)}$$

$$a + b - c = 0 \qquad \ldots\text{(iii)}$$

and $$-2a = -1 \qquad \ldots\text{(iv)}$$

Solving these three equations, we get

$$a = \frac{1}{2}, \quad c = -\frac{1}{2} \quad \text{and} \quad b = -1$$

Substituting these values in Eq. (i), we get

$$f = k(F)^{1/2}\,(l)^{-1}(\mu)^{-1/2}$$

or $$f = \frac{k}{l}\sqrt{\frac{F}{\mu}}$$

Experimentally, the value of k is found to be $\frac{1}{2}$.

Hence, $$f = \frac{1}{2l}\sqrt{\frac{F}{\mu}}$$

Limitations of Dimensional Analysis

The method of dimensions has the following limitations

(i) By this method the value of dimensionless constant cannot be calculated.

(ii) By this method the equation containing trigonometrical, exponential and logarithmic terms cannot be analysed.

(iii) If a physical quantity depends on more than three factors, then relation among them cannot be established because we can have only three equations by equalising the powers of M, L and T.

1.2 Significant Figures

Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement.

"All accurately known digits in a measurement plus the first uncertain digit together form significant figures."

Significant figures depends on the least count measuring instrument.

For example, when we measure the length of a straight line using a metre scale and it lies between 7.4 cm and 7.5 cm, we may estimate it as $l = 7.43$ cm. This expression has three significant figures out of these 7 and 4 are precisely known but the last digit 3 is only approximately known.

Rules for Counting Significant Figures

For counting significant figures, we use the following rules

Rule 1 All non-zero digits are significant. For example, $x = 2567$ has four significant figures.

Rule 2 The zeros appearing between two non-zero digits are counted in significant figures, no matter where the decimal point is, if any. For example, 6.028 has 4 significant figures.

Rule 3 If the number is less than 1, the zero(s) on the right of decimal point but to the left of first non-zero digit are not significant.

For example, 0.0042 has two significant digits.

Rule 4 The terminal or trailing zero(s) in a number without a decimal point are not significant. Thus, 426 m = 42600 cm = 426000 mm has three significant figures.

Rule 5 In a number with decimal, zeros to the right of last non-zero digit are significant.

For example, 4.600 or 0.002300 have four significant figures each.

Point of confusion and its remedy

Suppose we change the units,then we will write

$$\begin{aligned} 2.30 \text{ m} &= 23.0 \text{ cm} \\ &= 2300 \text{ mm} \\ &= 0.00230 \text{ km} \end{aligned}$$

When we are writing 2300 mm, then from Rule -4, we would conclude erroneously that the number has two significant figures, while in fact it has three significant figures and a mere change of units cannot change the number of significant figures.

To remove such ambiguities in determining the number of significant figures, apply following rule.

Rule 6 The power of 10 is irrelevant to the determination of significant figures. For example, in the measurements

$$\begin{aligned} 2.30 \text{ m} &= 2.30 \times 10^2 \text{ cm} \\ &= 2.30 \times 10^3 \text{ mm} \\ &= 2.30 \times 10^{-3} \text{ km} \end{aligned}$$

The significant figures are three in each measurement, because all zeros appearing in the base number in the scientific notation (in the power of 10) are significant.

Rule 7 A choice of change of different units does not change the number of significant digits or figures in a measurement.

For example, the length 7.03 cm has three significant figures. But in different units, the same value can be written as, 0.0703 m or 70.3 mm. All these measurements have the same number of significant figures (digits 7, 0 and 3) namely three.

This shows that location of decimal point is of no consequence in determining the number of significant figures.

Measured value	Number of significant figures	Rule
12376	5	1
6024.7	5	2
0.071	2	3
410 m	3	4
2.40	3	6
1.6×10^{10}	2	7

Rounding off a Digit

Following are the rules for rounding off a measurement

Rule 1 If the number lying to the right of cut-off digit is less than 5, then the cut-off digit is retained as such. However, if it is more than 5, then the cut-off digit is increased by 1.

For example, $x = 6.24$ is rounded off to 6.2 to two significant digits and $x = 5.328$ is rounded off to 5.33 to three significant digits.

Rule 2 If the insignificant digit to be dropped is 5 then the rule is

(i) it the preceding digit is even, the insignificant digit is simply dropped.

(ii) if the preceding digit is odd, the preceding digit is raised by 1.

For example, $x = 6.265$ is rounded off to $x = 6.26$ to three significant digits and, $x = 6.275$ is rounded off to $x = 6.28$ to three significant digits.

Algebraic Operations with Significant Figures

In addition , subtraction, multiplication or division the final result should not have more significant figures than the original data from which it was obtained. To understand this, let us consider a chain of which all links are strong except the one. The chain will obviously break at the weakest link. Thus, the strength of the chain cannot be more than the strength of the weakest link in the chain.

(i) **Addition and Subtraction** Suppose in the measured values to be added or subtracted, the least number of significant digits after the decimal is *n*. Then, in the sum or difference also, the number of significant digits after the decimal should be *n*.

Example $1.2 + 3.45 + 6.789 = 11.439 \approx 11.4$

Here, the least number of significant digits after the decimal is one. Hence, the result will be 11.4 (when rounded off to smallest number of decimal places).

Example $12.63 - 10.2 = 2.43 \approx 2.4$

(ii) **Multiplication or Division** Suppose in the measured values to be multiplied or divided, the least number of significant digits be *n*, then in the product or quotient, the number of significant digits should also be *n*.

Example $1.2 \times 36.72 = 44.064 \approx 44$

The least number of significant digits in the measured values are two. Hence, the result when rounded off to two significant digits become 44. Therefore, the answer is 44.

Example $\frac{1100}{10.2} = 107.8431373 \approx 110$

As 1100 has minimum number of significant figures (*i.e.*, 2), therefore the result should also contain only two significant digits. Hence, the result when rounded off to two significant digits becomes 110.

Example $\frac{1100 \text{ m/s}}{10.2 \text{ m/s}} = 107.8431373 \approx 108$

Note In this case answer becomes 108. Think why?

1.3 Error Analysis

No measurement is perfect, as the errors involved in a measurement cannot be removed completely. Measured value is always somewhat different from the true value. The difference is called an error.

Errors can be classified in two ways. First classification is based on the cause of error. Systematic errors and random errors fall in this group.

Second classification is based on the magnitude of error. Absolute error, mean absolute error and relative (or fractional) error lie on this group. Now let us discuss them separately.

(i) **Systematic errors** These are the errors whose causes are known to us. Such errors can therefore be minimised. Following are few causes of these errors.

(a) Instrumental errors may be due to erroneous instruments. These errors can be reduced by using more accurate instruments and applying zero correction, when required.

(b) Sometimes errors arise on account of ignoring certain facts. For example, in measuring time period of simple pendulum error may creap because no consideration is taken of air resistance. These errors can be reduced by applying proper corrections to the formula used.

(c) Change in temperature, pressure, humidity etc., may also sometimes cause errors in the result. Relevant corrections can be made to minimise their effects.

(ii) **Random errors** The causes of random errors are not known. Hence, it is not possible to remove them completely. These errors may arise due to a variety of reasons. For example, the reading of a sensitive beam balance may change by the vibrations caused in the building due to persons moving in the laboratory or vehicles running nearby. The random errors can be minimised by repeating the observation a large number of times and taking the arithmetic mean of all the observations. The mean value would be very close to the most accurate reading. Thus,

$$a_{\text{mean}} = \frac{a_1 + a_2 + \dots + a_n}{n}$$

(iii) **Absolute errors** The difference between the true value and the measured value of a quantity is called an absolute error. Usually the mean value a_m is taken as the true value. So, if

$$a_m = \frac{a_1 + a_2 + \dots + a_n}{n}$$

Then by definition, absolute errors in the measured values of the quantity are,

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
$$\dots \quad \dots \quad \dots$$
$$\Delta a_n = a_m - a_n$$

Absolute error may be positive or negative.

(iv) **Mean absolute error** It is the arithmetic mean of the magnitudes of absolute errors. Thus,

$$\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_2| + \dots + |\Delta a_n|}{n}$$

The final result of measurement can be written as

$$a = a_m \pm \Delta a_{\text{mean}}$$

This implies that value of a is likely to lie between $a_m + \Delta a_{\text{mean}}$ and $a_m - \Delta a_{\text{mean}}$.

(v) **Relative or fractional error** The ratio of mean absolute error to the mean value of the quantity measured is called relative or fractional error. Thus,

$$\text{Relative error} = \frac{\Delta a_{\text{mean}}}{a_m}$$

Relative error expressed in percentage is called as the percentage error, *i.e.*,

$$\text{Percentage error} = \frac{\Delta a_{\text{mean}}}{a_m} \times 100$$

Example 1.6 *The diameter of a wire as measured by a screw gauge was found to be 2.620, 2.625, 2.630, 2.628 and 2.626 cm. Calculate*

(a) mean value of diameter,
(b) absolute error in each measurement,
(c) mean absolute error,
(d) fractional error,
(e) percentage error, and
(f) express the result in terms of percentage error.

Solution (a) Mean value of diameter,

$$a_m = \frac{2.620 + 2.625 + 2.630 + 2.628 + 2.626}{5}$$
$$= 2.6258 \text{ cm} = 2.626 \text{ cm}$$

(rounding off to three decimal places)

(b) Taking a_m as the true value, the absolute errors in different observations are,

$$\Delta a_1 = 2.626 - 2.620 = +0.006 \text{ cm}$$
$$\Delta a_2 = 2.626 - 2.625 = +0.001 \text{ cm}$$
$$\Delta a_3 = 2.626 - 2.630 = -0.004 \text{ cm}$$
$$\Delta a_4 = 2.626 - 2.628 = -0.002 \text{ cm}$$
$$\Delta a_5 = 2.626 - 2.626 = 0.000 \text{ cm}$$

(c) Mean absolute error,

$$\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + |\Delta a_4| + |\Delta a_5|}{5}$$
$$= \frac{0.006 + 0.001 + 0.004 + 0.002 + 0.000}{5}$$
$$= 0.0026$$
$$= 0.003$$

(rounding off to three decimal places)

(d) Fractional error $= \pm \frac{\Delta a_{\text{mean}}}{a_m}$

$$= \pm \frac{0.003}{2.626}$$
$$= \pm 0.001$$

(e) Percentage error $= \pm 0.001 \times 100$

$$= \pm 0.1\%$$

(f) Diameter of wire can be written as,

$$d = 2.626 \text{ cm} \pm 0.1\%$$

Combination of Errors

(i) **Errors in sum or difference** Let $x = a \pm b$.

Further, let Δa is the absolute error in the measurement of a, Δb is the absolute error in the measurement of b and Δx is the absolute error in the measurement of x. Then,

$$x + \Delta x = (a \pm \Delta a) \pm (b \pm \Delta b)$$
$$= (a \pm b) \pm (\pm \Delta a \pm \Delta b)$$
$$= x \pm (\pm \Delta a \pm \Delta b)$$

or $\Delta x = \pm \Delta a \pm \Delta b$

The four possible values of Δx are $(\Delta a - \Delta b)$, $(\Delta a + \Delta b)$, $(-\Delta a - \Delta b)$ and $(-\Delta a + \Delta b)$. Therefore, the maximum absolute error in x is

$$\Delta x = \pm (\Delta a + \Delta b)$$

i.e., the maximum absolute error in sum and difference of two quantities is equal to sum of the absolute errors in the individual quantities.

Example 1.7 *The volumes of two bodies are measured to be* $V_1 = (10.2 \pm 0.02)\ cm^3$ *and* $V_2 = (6.4 \pm 0.01)\ cm^3$. *Calculate sum and difference in volumes with error limits.*

Solution $V_1 = (10.2 \pm 0.02)\ \text{cm}^3$

and $V_2 = (6.4 \pm 0.01)\ \text{cm}^3$

$$\Delta V = \pm (\Delta V_1 + \Delta V_2)$$

$$= \pm (0.02 + 0.01)\ \text{cm}^3 = \pm 0.03\ \text{cm}^3$$

$$V_1 + V_2 = (10.2 + 6.4)\ \text{cm}^3 = 16.6\ \text{cm}^3$$

and $V_1 - V_2 = (10.2 - 6.4)\ \text{cm}^3 = 3.8\ \text{cm}^3$

Hence, sum of volumes $= (16.6 \pm 0.03)\ \text{cm}^3$

and difference of volumes $= (3.8 \pm 0.03)\ \text{cm}^3$

(ii) **Errors in a product** Let $x = ab$

Then, $(x \pm \Delta x) = (a \pm \Delta a)(b \pm \Delta b)$

or $x\left(1 \pm \frac{\Delta x}{x}\right) = ab\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)$

or $1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \pm \frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$ (as $x = ab$)

or $\pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$

Here, $\frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$ is a small quantity, so can be neglected.

Hence, $\pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b}$

Possible values of $\frac{\Delta x}{x}$ are $\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$, $\left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$, $\left(-\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$.

Hence, maximum possible value of

$$\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$$

Therefore, maximum fractional error in product of two (or more) quantities is equal to sum of fractional errors in the individual quantities.

(iii) **Error in division** Let $x = \frac{a}{b}$

Then, $x \pm \Delta x = \frac{a \pm \Delta a}{b \pm \Delta b}$

or $x\left(1 \pm \frac{\Delta x}{x}\right) = \frac{a\left(1 \pm \frac{\Delta a}{a}\right)}{b\left(1 \pm \frac{\Delta b}{b}\right)}$

or $\left(1 \pm \frac{\Delta x}{x}\right) = \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)^{-1}$ $\left(\text{as } x = \frac{a}{b}\right)$

As $\frac{\Delta b}{b} << 1$, so expanding binomially, we get

$$\left(1 \pm \frac{\Delta x}{x}\right) = \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \mp \frac{\Delta b}{b}\right)$$

or $1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta a}{a} \mp \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$

Here, $\frac{\Delta a}{a} \cdot \frac{\Delta b}{b}$ is a small quantity, so can be neglected.

Hence, $\pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \mp \frac{\Delta b}{b}$

Possible values of $\frac{\Delta x}{x}$ are $\left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$, $\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$, $\left(-\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$. Therefore, the maximum value of

$$\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$$

Or, the maximum value of fractional error in division of two quantities is equal to the sum of fractional errors in the individual quantities.

(iv) **Error in quantity raised to some power**

Let $x = \frac{a^n}{b^m}$

Then, $\ln(x) = n \ln(a) - m \ln(b)$

Differentiating both sides, we get

$$\frac{dx}{x} = n\frac{da}{a} - m\frac{db}{b}$$

In terms of fractional error we may write,

$$\pm \frac{\Delta x}{x} = \pm n\frac{\Delta a}{a} \mp m\frac{\Delta b}{b}$$

Therefore, maximum value of

$$\frac{\Delta x}{x} = \pm\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right)$$

Example 1.8 *The mass and density of a solid sphere are measured to be* (12.4 ± 0.1) kg *and* $(4.6 \pm 0.2)\ kg/m^3$. *Calculate the volume of the sphere with error limits.*

Solution Here, $m \pm \Delta m = (12.4 \pm 0.1)$ kg

and $\rho \pm \Delta\rho = (4.6 \pm 0.2)\ \text{kg/m}^3$

$$\text{Volume } V = \frac{m}{\rho} = \frac{12.4}{4.6}$$

$$= 2.69\ \text{m}^3 = 2.7\ \text{m}^3$$

(rounding off to one decimal place)

Now, $\frac{\Delta V}{V} = \pm\left(\frac{\Delta m}{m} + \frac{\Delta\rho}{\rho}\right)$

or $\Delta V = \pm\left(\frac{\Delta m}{m} + \frac{\Delta\rho}{\rho}\right) \times V$

$$= \pm\left(\frac{0.1}{12.4} + \frac{0.2}{4.6}\right) \times 2.7$$

$$= \pm 0.14$$

$\therefore$ $V \pm \Delta V = (2.7 \pm 0.14)\ \text{m}^3$

Example 1.9 *Calculate percentage error in determination of time period of a pendulum*

$$T = 2\pi \sqrt{\frac{l}{g}}$$

where l and g are measured with ± 1% and ± 2% errors.

Solution $\frac{\Delta T}{T} \times 100 = \pm \left(\frac{1}{2} \times \frac{\Delta l}{l} \times 100 + \frac{1}{2} \times \frac{\Delta g}{g} \times 100 \right)$

$$= \pm \left(\frac{1}{2} \times 1 + \frac{1}{2} \times 2 \right) = \pm 1.5\%$$

Least Count

The minimum measurement that can be measured accurately by an instrument is called the least count. The least count of a metre scale graduated in millimetre mark is 1 mm. The least count of a watch having seconds hand is 1 s.

Key-Terms for Concepts

- Least count of vernier callipers

$$= \begin{Bmatrix} \text{Value of 1 part of} \\ \text{main scale } (s) \end{Bmatrix} - \begin{Bmatrix} \text{Value of one part on} \\ \text{vernier scale } (v) \end{Bmatrix}$$

Least count of vernier callipers = 1MSD − 1VSD

where, MSD = Main Scale Division

VSD = Vernier Scale Division

- Least count = $\frac{\text{Value of 1 part on main scale } (s)}{\text{Number of parts on vernier scale } (n)}$
- Least count of screw guage

$$= \frac{\text{Pitch } (p)}{\text{Number of parts on circular scale } (n)}$$

Solved Examples

Example 1. *Check the correctness of the relation* $s = ut + \frac{1}{2}at^2$ *where u is initial velocity, a the acceleration, t the time and s the displacement.*

Solution Writing the dimensions of either side of the given equation.

LHS = s = displacement = $[M^0LT^0]$

RHS = ut = velocity × time = $[M^0LT^{-1}][T] = [M^0LT^0]$

and $\frac{1}{2}at^2$ = (acceleration) × (time)^2

$$= [M^0LT^{-2}][T]^2 = [M^0LT^0]$$

As LHS = RHS, formula is dimensionally correct.

Example 2. *Write the dimensions of a and b in the relation,*

$$P = \frac{b - x^2}{at}$$

where P is power, x the distance and t the time.

Solution The given equation can be written as,

$$Pat = b - x^2$$

Now, $[Pat] = [b] = [x^2]$

or $[b] = [x^2] = [M^0L^2T^0]$

and $[a] = \frac{[x^2]}{[Pt]} = \frac{[L^2]}{[ML^2T^{-3}][T]}$

$$= [M^{-1}L^0T^2]$$

Example 3. *The centripetal force F acting on a particle moving uniformly in a circle may depend upon mass (m), velocity (v) and radius (r) of the circle. Derive the formula for F using the method of dimensions.*

Solution Let $F = k\,(m)^x\,(v)^y\,(r)^z$ ...(i)

Here, k is a dimensionless constant of proportinality.

Writing the dimensions of RHS and LHS in Eq. (i), we have

$$[MLT^{-2}] = [M]^x\,[LT^{-1}]^y\,[L]^z$$

$$= [M^xL^{y+z}\,T^{-y}]$$

Equating the powers of M, L and T of both sides, we have

$x = 1$, $y = 2$ and $y + z = 1$

or $z = 1 - y = -1$

Putting the values in Eq. (i), we get

$$F = kmv^2r^{-1} = k\frac{mv^2}{r}$$

$$F = \frac{mv^2}{r} \quad \text{(where } k = 1\text{)}$$

Example 4. *Write down the number of significant figures in the following :*

(a) 6428 *(b)* 62.00 *m*

(c) 0.00628 *cm* *(d)* 1200 *N*

Solution (a) 6428 has four significant figures.

(b) 62.00 m has four significant figures.

(c) 0.00628 cm has three significant figures.

(d) 1200 N has four significant figures.

Example 5. Round off to four significant figures :

(a) 45.689 *(b)* 2.0082

Solution (a) 45.69

(b) 2.008

Example 6. *Add* 6.75×10^3 *cm to* 4.52×10^2 *cm with regard to significant figures.*

Solution
$$a = 6.75 \times 10^3 \text{ cm}$$
$$b = 4.52 \times 10^2 \text{ cm} = 0.452 \times 10^3 \text{ cm} = 0.45 \times 10^3 \text{ cm}$$
(upto 2 places of decimal)
$$\therefore \quad a + b = (6.75 \times 10^3 + 0.45 \times 10^3) \text{ cm} = 7.20 \times 10^3 \text{ cm}$$

Example 7. *A thin wire has a length of* 21.7 *cm and radius* 0.46 *cm. Calculate the volume of the wire to correct significant figures.*

Solution Given $l = 21.7$ cm,
$$r = 0.46 \text{ mm} = 0.046 \text{ cm}$$
Volume of wire $V = \pi r^2 l$
$$= \frac{22}{7}(0.046)^2 (21.7)$$
$$= 0.1443 \text{ cm}^3 = 0.14 \text{ cm}^3$$

Note The result is rounded off to least number of significant figures in the given measurements *i.e.*, 2 (in 0.46 mm).

Example 8. *The refractive index (n) of glass is found to have the values* 1.49, 1.50, 1.52, 1.54 *and* 1.48. *Calculate*

(a) the mean value of refractive index,

(b) absolute error in each measurement,

(c) mean absolute error,

(d) fractional error, and

(e) percentage error.

Solution (a) Mean value of refractive index,
$$n_m = \frac{1.49 + 1.50 + 1.52 + 1.54 + 1.48}{5}$$
$$= 1.505 = 1.51$$
(rounded off to two decimal places)

(b) Taking n_m as the true value, the asbolute errors in different observations are,
$$\Delta n_1 = 1.51 - 1.49 = +0.02$$
$$\Delta n_2 = 1.51 - 1.50 = +0.01$$
$$\Delta n_3 = 1.51 - 1.52 = -0.01$$
$$\Delta n_4 = 1.51 - 1.54 = -0.03$$
$$\Delta n_5 = 1.51 - 1.48 = +0.03$$

(c) Mean absolute error,
$$\Delta n_{\text{mean}} = \frac{|\Delta n_1| + |\Delta n_2| + |\Delta n_3| + |\Delta n_4| + |\Delta n_5|}{5}$$
$$= \frac{0.02 + 0.01 + 0.01 + 0.03 + 0.03}{5}$$
$$= 0.02$$

(d) Fractional error $= \dfrac{\pm \Delta n_{\text{mean}}}{n_m} = \dfrac{\pm 0.02}{1.51}$
$$= \pm 0.0132$$

(e) Percentage error $= (\pm 0.0132 \times 100) = \pm 1.32\%$

Example 9. *The radius of sphere is measured to be* (2.1 ± 0.5) *cm. Calculate its surface area with error limits.*

Solution Surface area, $S = 4\pi r^2$
$$= (4)\left(\frac{22}{7}\right)(2.1)^2$$
$$= 55.44 = 55.4 \text{ cm}^2$$

Further, $$\frac{\Delta S}{S} = 2\frac{\Delta r}{r}$$

or $$\Delta S = 2\left(\frac{\Delta r}{r}\right)(S)$$
$$= \frac{2 \times 0.5 \times 55.4}{2.1}$$
$$= 26.38 = 26.4 \text{ cm}^2$$
$$\therefore \quad S = (55.4 \pm 26.4) \text{ cm}^2$$

Example 10. *Calculate focal length of a spherical mirror from the following observations. Object distance* $u = (50.1 \pm 0.5)$ *cm and image distance* $v = (20.1 \pm 0.2)$ *cm.*

Solution
$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

or $$f = \frac{uv}{u + v}$$
$$= \frac{(50.1)(20.1)}{(50.1 + 20.1)} = 14.3 \text{ cm}$$

Also, $$\frac{\Delta f}{f} = \pm\left[\frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta u + \Delta v}{u + v}\right]$$
$$= \pm\left[\frac{0.5}{50.1} + \frac{0.2}{20.1} + \frac{0.5 + 0.2}{50.1 + 20.1}\right]$$
$$= [0.00998 + 0.00995 + 0.00997]$$
$$= \pm(0.0299)$$
$$\therefore \quad \Delta f = 0.0299 \times 14.3$$
$$= 0.428 = 0.4 \text{ cm}$$
$$\therefore \quad f = (14.3 \pm 0.4) \text{ cm}$$

NCERT Corner

1. Fill in the blanks

(a) The volume of a cube of side 1 cm is equal to ... m^3.

(b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to...... mm^2.

(c) A vehicle moving with a speed of 18 kmh^{-1} covers...... m in 1s.

(d) The relative density of lead is 11.3. Its density is ...... g cm^{-3} or ......kg m^{-3}

Solution

(a) The volume of a cube of side 1 cm is given by,

$$V = (10^{-2}\ m)^3 = 10^{-6}\ m^3$$

(b) The surface area of a solid cylinder of radius r and height h is given by

A = Area of two circles + curved surface area

$$= 2\pi r^2 + 2\pi rh$$

$$= 2\pi r(r + h)$$

Here, $r = 2$ cm $= 20$ mm, $h = 10$ cm $= 100$ mm

$$\therefore \quad A = 2 \times \frac{22}{7} \times 20\ (20 + 100)\ (mm)^2$$

$$= 15099\ mm^2$$

$$= 1.5099 \times 10^4\ mm^2$$

$$= 1.5 \times 10^4\ mm^2$$

(c) Here $v = 18\ km\ h^{-1} = \frac{18 \times 1000\ m}{3600\ s} = 5\ ms^{-1}$

$$t = 1s$$

$$\therefore \quad x = vt = 5 \times 1 = 5m$$

(d) Relative density of lead = 11.3

Density of water = 1 g cm^{-3}

$$\text{Relative density of lead} = \frac{\text{density of lead}}{\text{density of water}}$$

$\therefore$ Density of lead

= relative density of lead × density of water

$$= 11.3 \times 1\ g\ cm^{-3}$$

$$= 11.3\ g\ cm^{-3}$$

In SI system density of water $= 10^3\ kg\ m^{-3}$

$\therefore$ Density of lead $= 11.3 \times 10^3\ kg\ m^{-3}$

$$= 1.13 \times 10^4\ kg\ m^{-3}$$

2. Fill in the blanks by suitable conversion of units.

(a) $1\ kg\ m^2\ s^{-2} =$...... g $cm^2\ s^{-2}$

(b) 1 m = ... ly (light year)

(c) 3.0 ms^{-2} =...... km h^{-2}

(d) $G = 6.67 \times 10^{11} Nm^2 kg^{-2}$ =...... $cm^3\ s^{-2}\ g^{-1}$

Solution

(a) $1\ kg\ m^2 s^{-2} = 1\ (10^3 g)\ (10^2 cm)^2 s^{-2}$

$$= 10^3 \times 10^4\ g\ cm^2 s^{-2} = 10^7\ g\ cm^2\ s^{-2}$$

(b) 1 light year (ly) $= 9.46 \times 10^{15}$ m

$$\therefore \quad 1m = \frac{1}{9.46 \times 10^{15}}\ ly = 1.057 \times^{-16}\ ly$$

(c) $3.0\ ms^{-2} = 3 \times 10^{-3}\ km \times \left(\frac{1}{60 \times 60}\ h^{-1}\right)^{-2}$

$$= 3 \times 10^{-3} \times (3600)^2\ kmh^{-2}$$

$$= 3.888 \times 10^4\ kmh^{-2} = 3.9 \times 10^4\ kmh^{-2}$$

(d) $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$

$$= 6.67 \times 10^{-11}\ (10^5 dyne)\ (10^2\ cm)^2\ (10^3\ g)^{-2}$$

$$= 6.67 \times 10^{-11} \times 10^5 \times 10^4 \times 10^{-6}\ dyne\ cm^2 g^{-2}$$

$$= 6.67 \times 10^{-8}\ (g\ cm\ s^{-2})\ cm^2\ g^{-2}$$

$$= 6.67 \times 10^{-8}\ cm^3\ g^{-1}\ s^{-2}$$

3. A calorie is a unit of heat or energy and it equals about 4.2 J, where $1J = 1kgm^2s^{-2}$. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length β m, and the unit of time is γ s. Show that a calorie has a magnitude $4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2$ in terms of new units.

Solution $n_1 u_1 = n_2 u_2$

or $$n_2 = n_1 \frac{u_1}{u_2} = n_1 \frac{[M_1^a\ L_1^b\ T_1^c]}{[M_2^a\ L_2^b\ T_2^c]}$$

$$= n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$1\ cal = 4.2\ J = 4.2\ kg\ m^2\ s^{-2},$$

$$\therefore \quad a = 1, b = 2, c = -2$$

$$\therefore \quad n_2 = 4.2\left[\frac{1\ kg}{\alpha\ kg}\right]^1 \left[\frac{1\ m}{\beta\ m}\right]^2 \left[\frac{1\ s}{\gamma\ s}\right]^{-2}$$

$$n_2 = 4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2$$

$\therefore$ 1 cal $= 4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2$ in new system.

4. A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the sun and the earth in terms of the new unit if light takes 8 min and 20s to cover this distance?

Solution We are given that velocity of light in vacuum, $c = 1$ new unit of length s^{-1}.

Time taken by light of sun of reach the earth, $t = 8$ min 20s.

$$= 8 \times 60 + 20 = 500\ s$$

$\therefore$ Distance between the sun and the earth,

$$x = c \times t$$

$$= 1\ \text{new unit of length}\ s^{-1} \times 500\ s$$

$$= 500\ \text{new units of length}$$

5. Which of the following is the most precise device for measuring length?

(a) A vernier callipers with 20 divisions on the sliding scale.

(b) A screw gauge of pitch 1 mm and 100 divisions on the circular scale.

(c) An optical instrument that can measure length to within a wavelength of light.

Solution The most precise device is that whose least count is minimum.

(a) Least count of vernier callipers

$= 1 \text{ MSD} - 1 \text{ VSD}$

$= 1 \text{ MSD} - \frac{19}{20} \text{ MSD} = \frac{1}{20} \text{ MSD}$

$= \frac{1}{20} \text{ mm}$

$= \frac{1}{200} \text{ cm}$

$= 0.005 \text{ cm}$

(b) Least count of screw gauge

$= \frac{\text{Pitch}}{\text{Number of divisions of circular scale}}$

$= \frac{1}{100} \text{ mm} = \frac{1}{1000} \text{ cm}$

$= 0.001 \text{ cm}$

(c) Wavelength of light, $\lambda \approx 10^{-5} \text{ cm} = 0.00001 \text{ cm}$

$\therefore$ Least count of optical instrument $= 0.00001$ cm

Thus, clearly the optical instrument is the most precise.

6. State the number of significant figures in the following

(a) 0.007 m^2 (b) 2.64×10^{24} kg

(c) 0.2370 g cm^{-3} (d) 6.320 J

(e) 6.032 Nm^{-2} (f) 0.0006032 m^2

Solution The number of significant figures is as given below.

(a) 1 (b) 3

(c) 4 (d) 4

(e) 4 (f) 4

7. The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

Solution Here length, $l = 4.234$ m

Breadth, $b = 1.005$ m

Thickness, $h = 0.0201 \text{ m} = 2.01 \text{ cm}$

Area of the sheet $= 2\,(lb + bh + hl)$

$= 2\,(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)$

$= 8.7209468 \text{ m}^2$

As the least number of significant figures in thickness is 3,

$\therefore$ Area $= 8.72 \text{ m}^2$

Volume $= l \times b \times h$

$= 4.234 \times 1.005 \times 0.0201 \text{ m}^3 = 0.0855 \text{ m}^3$

8. The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the mass of the pieces to correct significant figures?

Solution (a) Total mass $= (2.300 + 0.02015 + 0.02017)$ kg

$= 2.34032$ kg

As the least number of significant figures in the mass of box is 2, so maximum number of significant figures in the result can be 2.

$\therefore$ Total mass $= 2.3$ kg

(b) Difference in masses $= 20.17 - 20.15 = 0.02$ g

Since there are two significant figures, so the difference in masses to the correct significant figures is 0.02 g.

9. A physical quantity P is related to four observables a, b, c and d as follows $P = \frac{a^3 b^2}{\sqrt{cd}}$. The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2% respectively. What is the percentage error in the quantity P? If the value of P calculated using the above relation turns out to be 3.763, to what value should you round off the result?

Solution $P = \frac{a^3 b^2}{\sqrt{(c)}d}$

$\therefore$ Percentage error in P is given by

$$\frac{\Delta P}{P} \times 100 = 3\left(\frac{\Delta a}{a} \times 100\right) + 2\left(\frac{\Delta b}{b} \times 100\right) + \frac{1}{2}\left(\frac{\Delta c}{c} \times 100\right) + \left(\frac{\Delta d}{d} \times 100\right) \quad \ldots(i)$$

$$\left.\begin{aligned} \frac{\Delta a}{a} \times 100 = 1\%, \quad \frac{\Delta c}{c} \times 100 = 4\% \\ \frac{\Delta b}{b} \times 100 = 3\%, \quad \frac{\Delta d}{d} \times 100 = 2\% \end{aligned}\right] \quad \ldots(ii)$$

$\therefore$ From Eqs. (i) and (ii), we get

$$\frac{\Delta P}{P} \times 100 = 3 \times 1\% + 2 \times 3\% + \frac{1}{2} \times 4\% + 2\%$$

$$= 3 + 6 + 2 + 2 = 13\%$$

The calculation of error clearly shows that the number of significant figures is 2, so the result of P may be rounded off to two significant digits *i.e.* $P = 3.763 = 3.8$.

10. A book with many printing errors contains four different formulae for the displacement y of a particle under going a certain periodic motion :

(a) $y = a \sin \frac{2\pi t}{T}$

(b) $y = a \sin vt$

(c) $y = \frac{a}{T} \sin (t / a)$

(d) $y = \left(\frac{a}{\sqrt{2}}\right)\left(\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T}\right)$

(where a = maximum displacement of the particle, v = speed of the particle, T = time period of motion). Rule out the wrong formulae on dimensional grounds.

Solution The argument of a trigonometrical function *i.e.*, angle is dimensionless. Now here in each case dimensions of LHS is [L] and dimensions of RHS in

(a) = [L] $\left(\text{angle } \frac{2\pi t}{T} \text{ is dimensionless}\right)$

(b) = [L] sin $[LT^{-1}\ (T)]$ = [L] sin [L]

(angle is not dimensionless here)

(c) $= \frac{[L]}{[T]} \sin \frac{[T]}{[L]} = [LT^{-1}] \sin [TL^{-1}]$

(angle is not dimensionless here)

(d) $= [L]\left[\sin \frac{T}{T} + \cos \frac{T}{T}\right] = [L]$

$\therefore$ Formulae (b) and (c) are wrong.

11. The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å. 1 Å $= 10^{-10}$ m. The size of the hydrogen atom is about 0.5 Å. What is the total atomic volume in m^3 of a mole of hydrogen atoms?

Solution $r = 0.5Å = 0.5 \times 10^{-10}$ m

V_1 = Volume of each hydrogen atom $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times 3.14 \times (0.5 \times 10^{-10})^3$

$= 5.236 \times 10^{-31}$ m^3

According to Avogadro's hypothesis, one mole of hydrogen contains

$N = 6.023 \times 10^{23}$ atoms

$\therefore$ Atomic volume of 1 mole of hydrogen atoms,

$V = NV_1$

or $V = 6.023 \times 10^{23} \times 5.236 \times 10^{-31}$

$= 3.154 \times 10^{-7}$ m^3

$\cong 3 \times 10^{-7}$ m^3

12. One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen? (Take the size of hydrogen molecule to be about 1 Å). Why is this ratio so large?

Solution d = diameter of hydrogen molecule = 1Å

Molar volume of one mole of hydrogen

$= 22.4$ L $= 22.4 \times 10^{-3}$ m^3

r = radius of one molecule of hydrogen

$= \frac{d}{2} = 0.5$ Å

$= 0.5 \times 10^{-10}$ m

Volume of one molecule of hydrogen

$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\,(0.5 \times 10^{-10})^3$

$= 5.236 \times 10^{-31}$ m^3

1 mole has 6.023×10^{23} atoms or molecules of H_2

$\therefore$ Atomic volume of one mole of hydrogen

$= 6.023 \times 10^{23} \times 5.236 \times 10^{-31}$ m^3

$= 3.154 \times 10^{-7}$ m^3

$$\therefore \frac{\text{Molar volume}}{\text{Atomic volume}} = \frac{22.4 \times 10^{-3}\ \text{m}^3}{3.154 \times 10^{-7}\ \text{m}^3}$$

$= 7.1 \times 10^4 = 7 \times 10^4$

The large value of the ratio shows that the inter molecular separation in a gas in much larger than size of a mulecule.

13. The nearest star to our solar system is 4.29 light years away. How much is this distance in terms of parsecs?

Solution Distance = 4.29 light year

$= 4.29 \times 9.46 \times 10^{15}$ m

($\because$ 1 ly $= 9.46 \times 10^{15}$ m)

$= \frac{4.29 \times 9.46 \times 10^{15}}{3.08 \times 10^{16}}$ parsec

($\because$ 1 parsec $= 3.08 \times 10^{16}$ m)

$= 1.318$ parsec $= 1.32$ parsec

14. It is claimed that the two cesium clocks, if allowed of run for 100 yr, free from any disturbance, may differ by only about 0.02s. What does this imply for the accuracy of the standard cesium clock in measuring a time interval of 1s?

Solution Time interval = 100 years

$= 100 \times 365 \times 24 \times 60 \times 60$ s

$= 3.155 \times 10^9$ s

Difference in time = 0.2s

$$\therefore \text{Fractional error} = \frac{\text{Difference in time(s)}}{\text{Time interval(s)}}$$

$= \frac{0.2}{3.155 \times 10^9}$

$= 6.34 \times 10^{-12}$

$= 10 \times 10^{-12} \approx 10^{-11}$

$\therefore$ In 1s, the difference is 10^{-11} to 6.34×10^{12}

Hence degree of accuracy shown by the atomic clock in 1s is 1 part in $\frac{1}{10^{-11}}$ to $\frac{1}{6.34 \times 10^{-12}}$ or 10^{11} to 10^{12}.

15. Estimate the average mass density of sodium atom assuming, its size to be about 2.5 Å (Use the known values of Avogadro's number, and the atomic mass of sodium). Compare it with the density of sodium in its crystalline phase 970 kg m^{-3}. Are the two densities of the same order of magnitude? If so, why?

Solution Average radius of sodium atom,

$r = 2.5$ Å $= 2.5 \times 10^{-10}$ m

$\therefore$ Volume of sodium atom $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times 3.14 \times (2.5 \times 10^{-10})^3$

$= 65.42 \times 10^{-30}$ m^3

Mass of a mole of sodium = 23 g = 23×10^{-3} kg.

One mole contains 6.023×10^{23} atoms, hence the mass of sodium atom,

$$M = \frac{23 \times 10^{-3}}{6.023 \times 10^{23}}\ \text{kg} = 3.82 \times 10^{-26}\ \text{kg}$$

$\therefore$ Average mass density of sodium atom,

$$\rho = \frac{M}{V} = \frac{3.82 \times 10^{-26}}{65.42 \times 10^{-30}}\ \text{kgm}^{-3}$$

$= 0.64 \times 10^3$ kgm^{-3}

Density of sodium in crystalline phase = 970 kgm^{-3}

$= 0.970 \times 10^3$ kgm^{-3}

$$\therefore \frac{\text{Average mass density of sodium atom}}{\text{Density of sodium of crystalline phase}} = \frac{0.64 \times 10^3}{0.970 \times 10^3}$$

$= 0.66$

Both densities are of the same order *i.e.* of the order of 10^3. This is because in the solid phase atoms are tightly packed, so the atomic mass density is close to the mass density of the solid.

16. A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine? (Speed of sound in water = 1450 m s^{-1}).

Solution Time taken by the wave to go from submarine to enemy submarine is

$$t = \frac{77}{2} = 38.5\,\text{s}$$

Speed of sound, $v = 1450\ \text{ms}^{-1}$

Distance of enemy submarine,

$$\therefore \qquad S = vt = 1450 \times 38.50$$
$$= 55825\ \text{m} = 55.825\ \text{km}$$

17. The farthest objects in our universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the earth. These objects (known as quasars) have many puzzling features which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us?

Solution Time taken, $t = 3 \times 10^9$ years

$$= 3 \times 10^9 \times 365 \times 24 \times 60 \times 60\,\text{s}$$

Velocity of light, $c = 3 \times 10^8\ \text{ms}^{-1}$

$\therefore$ Distance of quasar from earth $= ct$

$$= 3 \times 10^8 \times 3 \times 10^9 \times 365 \times 24 \times 3600\ \text{m}$$
$$= 2.8 \times 10^{25}\ \text{m}$$
$$= 2.8 \times 10^{22}\ \text{km}$$

Objective Problems (Level 1)

Measurement of Length and Time

1. Which one is not a unit of time?
(a) Leap year (b) Year
(c) Shake (d) Light year

2. "Parsec" is the unit of
(a) time (b) distance
(c) frequency (d) angular acceleration

3. One light-year distance is equal to
(a) 9.46×10^{10} km (b) 9.46×10^{12} km
(c) 9.46×10^{9} km (d) 9.46×10^{15} km

4. Parallactic second is equal to
(a) 9.4605×10^{15} m (b) 3.07×10^{16}m
(c) 1.496×10^{11}m (d) 3×10^{8} m

5. A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the sun and the earth in terms of the new unit, if light takes 8 min and 20 s to cover this distance?
(a) 300 (b) 400
(c) 500 (d) 600

6. A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair is 3.5 mm. What is the estimate on the thickness of the hair?
(a) 0.0035 mm (b) 0.035 mm
(c) 0.01 m (d) 0.7 mm

7. Which of the following is the most precise device for measuring length?
(a) a vernier calipers with 20 divisions on the sliding scale
(b) an optical instrument that can measure length to within a wavelength of light
(c) a screw gauge of pitch 1 mm and 100 divisions on the circular scale
(d) All the above are equally precise

Units and Dimensions

8. The dimensions of impulse are equal to that of
(a) force (b) linear momentum
(c) pressure (d) angular momentum

9. In the SI system, the unit of temperature is
(a) degree centigrade (b) kelvin
(c) degree celsius (d) degree fahrenheit

10. Which one of the following have same dimensions?
(a) Torque and force
(b) Potential energy and force
(c) Torque and potential energy
(d) Planck's constant and linear momentum

11. Which of the following does not posses the same dimensions as that of pressure?
(a) Stress (b) Bulk modulus
(c) Thrust (d) Energy density

12. Which of the following is a dimensional constant?
(a) Poission's ratio (b) Refractive index
(c) Relative density (d) Gravitational constant

13. Which one of the following is not the dimensionless quantity?
(a) Planck's constant (b) Dielectric constant
(c) Solid angle (d) Strain

14. Joule $\times$ second is the unit of
(a) energy (b) momentum
(c) angular momentum (d) power

15. Which of the following is not equal to watt?
(a) joule/second (b) ampere $\times$ volt
(c) $(\text{ampere})^2 \times \text{ohm}$ (d) ampere/volt

16. Which of the following is not the units of surface tension?
(a) N / m (b) J / m^2
(c) kg / s^2 (d) None of these

17. Wb/m^2 is equal to
(a) dyne (b) tesla (c) watt (d) henry

18. Dimensional formula for electromotive force is same as that for
(a) potential (b) current
(c) force (d) energy

19. Which of the following has the dimensions of pressure?
(a) $[ML^{-2}T^{-2}]$ (b) $[M^{-1}L^{-1}]$
(c) $[MLT^{-2}]$ (d) $[ML^{-1}T^{-2}]$

20. Dimensions of torque are
(a) $[M^2L^2T^2]$ (b) $[ML^2T^{-2}]$
(c) $[ML^0T^{-1}]$ (d) $[ML^2T^{-1}]$

21. Dimensions of impulse are
(a) $[ML^{-2}T^{-3}]$ (b) $[ML^{-2}]$
(c) $[MLT^{-1}]$ (d) $[MLT^{-2}]$

22. What is the dimensional formula of gravitational constant?
(a) $[ML^2T^{-2}]$ (b) $[ML^{-1}T^{-1}]$
(c) $[M^{-1}L^3T^{-2}]$ (d) None of these

23. Dimensions of surface tension are
(a) $[M^2L^2T^{-2}]$ (b) $[M^2LT^{-2}]$
(c) $[MT^{-2}]$ (d) $[MLT^{-2}]$

24. The dimensional formula for Young's modulus is
(a) $[ML^{-1}T^{-2}]$ (b) $[M^0LT^{-2}]$
(c) $[MLT^{-2}]$ (d) $[ML^2T^{-2}]$

25. Which of the following is the dimensions of the coefficient of friction?
(a) $[M^2L^2T]$ (b) $[M^0L^0T^0]$
(c) $[ML^2T^{-2}]$ (d) $[M^2L^2T^{-2}]$

26. The dimensional formula for the action will be
(a) $[MLT^{-2}]$ (b) $[M^2LT^{-2}]$
(c) $[ML^2T^{-1}]$ (d) $[M^2L^2T^{-2}]$

27. $[ML^{-1}T^{-1}]$ stand for dimensions of
(a) work (b) torque
(c) linear momentum (d) coefficient of viscosity

28. Dimensions of relative density is
(a) $[ML^{-2}]$ (b) $[ML^{-3}]$
(c) dimensionless (d) $[M^2L^{-6}]$

29. The dimensions of the ratio of angular to linear momentum us
(a) $[M^0LT^0]$ (b) $[MLT^{-1}]$
(c) $[ML^2T^{-1}]$ (d) $[M^{-1}L^{-1}T^{-1}]$

30. The dimensional formula for thermal resistance is
(a) $[ML^2T^{-3}K^{-1}]$ (b) $[ML^2T^{-2}A^{-1}]$
(c) $[ML^2T^{-3}K^{-2}]$ (d) $[M^{-1}L^{-2}T^3K]$

31. $[ML^2T^{-3}A^{-1}]$ is the dimensional formula for
(a) capacitance (b) resistance
(c) resistivity (d) potential difference

32. Temperature can be expressed as a derived quantity in terms of any of the following
(a) length and mass (b) mass and time
(c) length, mass and time (d) None of these

33. Given that $y = a\cos\left(\frac{t}{p} - qx\right)$, where t represents time in following statements is true?
(a) The unit of x is same as that of q
(b) The unit of x is same as that of p
(c) The unit of t is same as that of q
(d) The unit of t is same as that of p

34. The dimensional formula $[ML^0T^{-3}]$ is more closely associated with
(a) power (b) energy
(c) intensity (d) velocity gradient

35. Which of the following is dimensionally correct?
(a) Pressure = energy per unit area
(b) Pressure = energy per unit volume
(c) Pressure = force per unit volume
(d) Pressure = momentum per unit volume per unit time

36. Assuming that the mass m of the largest stone that can be moved by a flowing river depends upon the velocity v of the water, its density ρ and the acceleration due to gravity g. Then, m is directly proportional to
(a) v^3 (b) v^4
(c) v^5 (d) v^6

37. If p represents radiation pressure, c represent speed of light and Q represents radiation energy striking a unit area per second, then non-zero integers x, y and z such that $p^xQ^yc^z$ is dimensionless are
(a) $x = 1, y = 1, z = -1$
(b) $x = 1, y = -1, z = 1$
(c) $x = -1, y = 1, z = 1$
(d) $x = 1, y = 1, z = 1$

38. The units of length, velocity and force are doubled. Which of the following is the correct change in the other units?
(a) Unit of time is doubled
(b) Unit of mass is doubled
(c) Unit of momentum is doubled
(d) Unit of energy is doubled

39. Which of the following pairs has the same units?
(a) Wavelength and Rydberg constant
(b) Relative velocity and relative density
(c) Thermal capacity and Boltzmann constant
(d) Time period and acceleration gradient

40. The dimensional representation of specific resistance in terms of charge Q is
(a) $[ML^3T^{-1}Q^{-2}]$ (b) $[ML^2T^{-2}Q^2]$
(c) $[MLT^{-2}Q^{-1}]$ (d) $[ML^2T^{-2}Q^{-1}]$

41. Which of the following will have the dimensions of time?
(a) LC (b) $\frac{R}{L}$
(c) $\frac{L}{R}$ (d) $\frac{C}{L}$

42. If C and R denote capacity and resistance, the dimensions of CR are
(a) $[M^0L^0T]$
(b) $[ML^0T]$
(c) $[M^0L^0T^2]$
(d) not expressible in terms of M, L and T

43. The force F on a sphere of radius a moving in a medium with velocity v is given by $F = 6\pi\,\eta a\,v$. The dimensions of η are
(a) $[ML^{-3}]$ (b) $[MLT^{-2}]$
(c) $[MT^{-1}]$ (d) $[ML^{-1}T^{-1}]$

44. The equation of a wave is given by $y = a\sin\omega\left(\frac{x}{v} - k\right)$ where ω is angular velocity and v is the linear velocity. The dimension of k will be
(a) $[T^{-2}]$ (b) $[T^{-1}]$
(c) $[T]$ (d) $[LT]$

45. A force is given by $F = at + bt^2$, where t is the time. The dimensions of a and b are
(a) $[MLT^{-4}]$ and $[MLT]$ (b) $[MLT^{-1}]$ and $[MLT^0]$
(c) $[MLT^{-3}]$ and $[MLT^{-4}]$ (d) $[MLT^{-3}]$ and $[MLT^0]$

46. The dimensional formula for Planck's constant and angular momentum is
(a) $[ML^2T^{-2}]$ and $[MLT^{-1}]$ (b) $[ML^2T^{-1}]$ and $[ML^2T^{-1}]$
(c) $[ML^3T^{-1}]$ and $[ML^2T^{-2}]$ (d) $[MLT^{-1}]$ and $[MLT^{-2}]$

47. The dimension of $\frac{1}{2}\varepsilon_0 E^2$($\varepsilon_0$ is the permittivity of the space and E is electric field), is
(a) $[ML^2T^{-1}]$ (b) $[ML^{-1}T^{-2}]$ (c) $[ML^2T^{-2}]$ (d) $[MLT^{-1}]$

48. The dimensions of $\frac{a}{b}$ in the equation $p = \frac{a - t^2}{bx}$, where p is pressure, x is distance and t is time, are
(a) $[M^2LT^{-3}]$ (b) $[MT^{-2}]$
(c) $[LT^{-3}]$ (d) $[ML^3T^{-1}]$

49. Dimension of velocity gradient is
(a) $[M^0L^0T^{-1}]$ (b) $[ML^{-1}T^{-1}]$
(c) $[M^0LT^{-1}]$ (d) $[ML^0T^{-1}]$

50. The dimensional formula for emf e is MKS system will be
(a) $[ML^2T^{-2}Q^{-1}]$ (b) $[ML^2T^{-1}]$
(c) $[ML^{-2}Q^{-1}]$ (d) $[MLT^{-2}Q^{-2}]$

51. The velocity v of a particle at time t is given by $v = at + \frac{b}{t + c}$, where a, b and c are constants. The dimensions of a, b and c are respectively
(a) $[LT^{-2}]$, [L] and [T] (b) $[L^2]$, [T] and $[LT^2]$
(c) $[LT^2]$, [LT] and [L] (d) [L], [LT] and $[T^2]$

52. What is the units of $k = \frac{1}{4\pi\varepsilon_0}$?
(a) $C^2N^{-1}m^{-2}$ (b) Nm^2C^{-2}
(c) Nm^2C^2 (d) Unitless

53. Pressure gradient has the same dimensions as that of
(a) velocity gradient (b) potential gradient
(c) energy gradient (d) None of these

54. The unit of permittivity of free space, ε_0 is
(a) coulomb/newton-metre
(b) newton-metre^2/coulomb^2
(c) coulomb^2/newton-metre^2
(d) coulomb^2/(newton-metre)^2

55. Dimensions of electrical resistance are
(a) $[ML^2T^{-3}A^{-1}]$ (b) $[ML^2T^{-3}A^{-2}]$
(c) $[ML^3T^{-3}A^{-2}]$ (d) $[ML^{-1}L^3T^3A^3]$

56. The magnetic moment has dimensions of
(a) [LA] (b) $[L^2A]$
(c) $[LT^{-1}A]$ (d) $[L^2T^{-1}A]$

57. The dimensional representation of specific resistance in terms of charge Q is
(a) $[ML^3T^{-1}Q^{-2}]$ (b) $[ML^2T^{-2}Q^2]$
(c) $[MLT^{-2}Q^{-1}]$ (d) $[ML^2T^{-2}Q^{-1}]$

Significant Figures

58. The significant figures of the number 6.0023 is
(a) 2 (b) 5
(c) 4 (d) 1

59. What is the number of significant figures in 0.0310×10^3?
(a) 2 (b) 3
(c) 4 (d) 6

60. The number of significant figures in 11.118×10^{-6} V is
(a) 3 (b) 4
(c) 5 (d) 6

61. In which of the following numerical values, all zeros are significant?
(a) 0.2020 (b) 20.2
(c) 2020 (d) None of these

62. A student measured the diameter of a wire using a screw gauge with least count 0.001 cm and listed the measurements. The correct measurement is
(a) 8.320 cm (b) 5.3 cm
(c) 5.32 cm (d) 5.3200 cm

63. The length, breadth and thickness of rectangular sheet of metal are 4.234 m, 1.005 m and 2.01 cm respectively. The volume of the sheet to correct significant figures is
(a) 0.0855 m^3 (b) 0.086 m^3
(c) 0.08556 m^3 (d) 0.08 m^3

64. Three measurements are made as 18.425 cm, 7.21 cm and 5.0 cm. The addition should be written as
(a) 30.635 cm (b) 30.64 cm
(c) 30.63 cm (d) 30.6 cm

65. Subtract 0.2 J from 7.26 J and express the result with correct number of significant figures
(a) 7.1 (b) 7.06
(c) 7 (d) None of these

66. Multiply 107.88 by 0.610 and express the result with correct number of significant figures
(a) 65.8068 (b) 64.807
(c) 65.81 (d) 65.8

67. When 97.52 is divided by 2.54, the correct result is
(a) 38.3937 (b) 38.394
(c) 65.81 (d) 38.4

68. The radius of a thin wire is 0.16 mm. The area of cross-section of the wire in mm^2 with correct number of significant figures is
(a) 0.08 (b) 0.080
(c) 0.0804 (d) 0.080384

69. What is the number of significant figure in $(3.20 + 4.80) \times 10^5$?
(a) 5 (b) 4
(c) 3 (d) 2

70. What is the value of $[(5.0 \times 10^{-6})(5.0 \times 10^{-8})]$ with due regards to significant digits?
(a) 25×10^{-14} (b) 25.0×10^{-14}
(c) 2.50×10^{-13} (d) 250×10^{-15}

71. The mass of a box is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. The total mass of the box to correct significant figures is
(a) 2.3 kg (b) 2.34 kg (c) 2.3432 kg (d) 2.31 kg

72. Subtract 0.2 kg from 34 kg. The result in terms of proper significant figure is
(a) 33.8 kg (b) 33.80 kg
(c) 34 kg (d) 34.0 kg

73. The length, breadth and thickness of a block are given by $l = 12$ cm, $b = 6$ cm and $t = 2.45$ cm. The volume of the block according to the idea of significant figures should be
(a) $1 \times 10^2 cm^3$ (b) $2 \times 10^2 cm^3$
(c) $1.763 \times 10^2 cm^3$ (d) None of these

Error Analysis

74. The length of a rod is (11.05 ± 0.2) cm. What is the length of the two rods?
(a) (22.1 ± 0.05) cm (b) (22.1 ± 0.1) cm
(c) (22.10 ± 0.05) cm (d) (22.10 ± 0.2) cm

75. The radius of a ball is (5.2 ± 0.2) cm. The percentage error in the volume of the ball is approximately
(a) 11% (b) 4% (c) 7% (d) 9%

76. A physical quantity Q is calculated according to the expression

$$Q = \frac{A^3 B^3}{C\sqrt{D}}$$

If percentage errors in A, B, C, D are 2%, 1%, 3% and 4% respectively. What is the percentage error in Q?
(a) ± 8% (b) ± 10%
(c) ± 14% (d) ± 12%

77. A body travels uniformly a distance of (13.8 ± 0.2) m in a time (4.0 ± 0.3) s. The velocity of the body within error limit is
(a) (3.45 ± 0.2) ms^{-1}
(b) (3.45 ± 0.3) ms^{-1}
(c) (3.45 ± 0.4) ms^{-1}
(d) (3.45 ± 0.5) ms^{-1}

78. If the error in the measurement of momentum of a particle is (+ 100%), then the error in the measurement of kinetic energy is
(a) 100% (b) 200%
(c) 300% (d) 400%

79. If error in measuring diameter of a circle is 4%, the error in measuring radius of the circle would be
(a) 2% (b) 8%
(c) 4% (d) 1%

80. The values of two resistors are (5.0 ± 0.2) kΩ and (10.0 ± 0.1) kΩ. What is the percentage error in the equivalent resistance when they are connected in parallel?
(a) 2% (b) 5%
(c) 7% (d) 10%

81. The heat generated in a wire depends on the resistance, current and time. If the error in measuring the above are 1%, 2% and 1% respectively. The maximum error in measuring the heat is
(a) 8% (b) 6%
(c) 18% (d) 12%

82. A force F is applied on a square plate of side L. If the percentage error in the determination of L is 2% and that in F is 4%. What is the permissible error in pressure?
(a) 8% (b) 6%
(c) 4% (d) 2%

83. A cuboid has volume $V = l \times 2l \times 3l$, where l is the length of one side. If the relative percentage error in the measurement of l is 1%, then the relative percentage error in measurement of V is
(a) 18% (b) 6% (c) 3% (d) 1%

Miscellaneous Problems

84. The ratio of the SI unit to the CGS unit of modulus of rigidity is
(a) 10^2 (b) 10^{-2} (c) 10^{-1} (d) 10

85. Imagine a system of unit in which the unit of mass is 10 kg, length is 1 km and time is 1 min. Then, 1 J in this system is equal to
(a) 360 (b) 3.6
(c) 36×10^5 (d) 36×10^{-5}

86. The dimensional formula for molar thermal capacity is same as that of
(a) gas constant (b) specific heat
(c) Boltzmann's constant (d) Stefan's constant

87. In measuring electric energy, 1kWh is equal to
(a) 3.6×10^4 J (b) 3.6×10^6 J
(c) 7.3×10^6 J (d) None of these

88. Out of the following four dimensional quantities, which one qualifies to be called a dimensional constant?
(a) Acceleration due to gravity
(b) Surface tension of water
(c) Weight of a standard kilogram mass
(d) The velocity of light in vacuum

89. The square root of the product of inductance and capacitance has the dimensions of
(a) length (b) time
(c) mass (d) no dimension

90. With usual notation, the following equation, said to give the distance covered in the nth second. *i.e.*, $S_n = u + a\frac{(2n-1)}{2}$ is
(a) numerically correct only
(b) dimensionally correct only
(c) both dimensionally and numerically only
(d) neither numerically nor dimensionally correct

Objective Problems (Level 2)

1. A quantity is given by $X = \frac{\varepsilon_0 lV}{t}$, where V is the potential difference and l is the length. Then, X has dimensional formula same as that of
(a) resistance (b) charge
(c) voltage (d) current

2. The length of a strip measured with a metre rod is 10.0 cm. Its width measured with a vernier calipers is 1.00 cm. The least count of the metre rod is 0.1 cm and that of vernier calipers 0.01 cm. What will be error in its area?
(a) ± 13% (b) ± 7%
(c) ± 4% (d) ± 2%

3. The length of cylinder is measured with a metre rod having least count 0.1 cm. Its diameter is measured with vernier calipers having least count 0.01 cm. Given that length is 5.0 cm and radius is 2.0 cm. The percentage error in the calculated value of the volume will be
(a) 1.5% (b) 2.5% (c) 3.5% (d) 4%

4. The random error in the arithmetic means of 100 observations is x, then random error in the arithmetic mean of 400 observation would be

(a) $4x$ (b) $\frac{1}{4}x$
(c) $2x$ (d) $\frac{1}{2}x$

5. Dimensions of 'ohm' are same as
(a) $\frac{h}{e}$ (b) $\frac{h^2}{e}$ (c) $\frac{h}{e^2}$ (d) $\frac{h^2}{e^2}$
(where h is Planck's constant and e is charge)

6. Given that $\int \frac{dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1}\left[\frac{x-a}{a}\right]$
where a = constant. Using dimensional analysis, the value of n is
(a) 1 (b) zero
(c) -1 (d) None of these

7. If E = energy, G = gravitational constant, I = impulse and M = mass, then dimensions of $\frac{GIM^2}{E^2}$ are same as that of
(a) time (b) mass
(c) length (d) force

8. The dimensional formula for magnetic flux is
(a) $[ML^2T^{-2}A^{-1}]$ (b) $[ML^3T^{-2}A^{-2}]$
(c) $[M^0L^{-2}T^{-2}A^{-2}]$ (d) $[ML^2T^{-1}A^2]$

9. Using mass (M), length (L), time (T) and current (A) as fundamental quantities, the dimension of permeability is
(a) $[M^{-1}LT^{-2}A]$ (b) $[ML^{-2}T^{-2}A^{-1}]$
(c) $[MLT^{-2}A^{-2}]$ (d) $[MLT^{-1}A^{-1}]$

10. Let g be the acceleration due to gravity at earth's surface and K the rotational kinetic energy of the earth. Suppose the earth's radius decreases by 2%. Keeping mass to be constant, then
(a) g increases by 2% and K increases by 2%
(b) g increases by 4% and K increases by 4%
(c) g increases by 4% and K increases by 2%
(d) g increases by 2% and K increases by 4%

11. If the energy (E), velocity (v) and force (F) be taken as fundamental quantities, then the dimension of mass will be
(a) Fv^{-2} (b) Fv^{-1}
(c) Ev^{-2} (d) Ev^2

12. In a system of units, the units of mass, length and time are 1 quintal, 1 km and 1 h respectively. In this system 1 N force will be equal to
(a) 1 new unit (b) 129.6 new unit
(c) 427.6 new unit (d) 60 new unit

13. If force F, length L and time T are taken as fundamental units, the dimensional formula for mass will be
(a) $[FL^{-1}T^2]$ (b) $[FLT^{-2}]$
(c) $[FL^{-1}T^{-1}]$ (d) $[FL^5T^2]$

14. Given that $y = A\sin\left[\left(\frac{2\pi}{\lambda}(ct - x)\right)\right]$, where y and x are measured in metre. Which of the following statements is true?
(a) The unit of λ is same as that of x and A
(b) The unit of λ is same as that of x but not of A
(c) The unit of c is same as that of $\frac{2\pi}{\lambda}$
(d) The unit of $(ct - x)$ is same as that of $\frac{2\pi}{\lambda}$

15. The frequency of vibration of string is given by $f = \frac{p}{2l}\left[\frac{F}{m}\right]^{1/2}$. Here, p is number of segments in the string and l is the length. The dimensional formula for m will be
(a) $[M^0LT^{-1}]$ (b) $[ML^0T^{-1}]$
(c) $[ML^{-1}T^0]$ (d) $[M^0L^0T^0]$

Assertion and Reason

Directions (Q. Nos. 1-17) *These questions consists of two statements each printed as Assertion and Reason. While answering these questions you are required to choose any one of the following five responses.*
(a) If both Assertion and Reason are correct and Reason in the correct explanation of Assertion.
(b) If both Assertion and Reason are correct but Reason is not correct explanation of Assertion.
(c) If Assertion is true but Reason is false.
(d) If Assertion is false but Reason is true.
(e) If both Assertion and Reason are false.

1. **Assertion** Pressure has the dimensions of energy density.
Reason Energy density $= \frac{\text{energy}}{\text{volume}} = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$
= pressure.

2. **Assertion** Method of dimension cannot be used for deriving formulae containing trigonometrical ratios.
Reason This is because trigonometrical ratios have no dimensions.

3. **Assertion** When percentage errors in the measurement of mass and velocity are 1% and 2% respectively, the percentage error in KE is 5%.
Reason KE or $E = \frac{1}{2}mv^2$, $\frac{\Delta E}{E} = \frac{\Delta m}{m} + \frac{2\Delta v}{v}$

4. **Assertion** The error in the measurement of radius of the sphere is 0.3%. The permissible error in its surface area is 0.6%.
Reason The permissible error is calculated by the formula
$$\frac{\Delta A}{A} = 4\frac{\Delta r}{r}.$$

5. **Assertion** The light year and wavelength consist of dimensions of length.
Reason Both light year and wavelength represent time.

6. **Assertion** Number of significant figures in 0.005 is one and that in 0.500 are three.
Reason This is because zeros before decimal are non significant.

7. **Assertion** Out of two measurements $l = 0.7$ m and $l = 0.70$ m, the second one is more accurate.

Reason In every measurement, more the last digit is not accurately known.

8. Assertion When we change the unit of measurement of a quantity, its numerical value changes.
Reason Smaller the unit of measurement smaller is its numerical value.

9. Assertion L/R and CR both have same dimensions.
Reason L/R and CR both have dimension of time.

10. Assertion $\sqrt{\frac{\text{Magnetic dipole moment} \times \text{moment induction}}{\text{Moment of inertia}}}$
Dimensional formula $[M^0L^0T]$
Reason The given dimension is that of frequency.

11. Assertion $\sqrt{\frac{\text{Modulus of elasticity}}{\text{Density}}}$ has the unit ms^{-1}.
Reason Acceleration has the dimensions of $\frac{1}{(\sqrt{\varepsilon_0 \mu_0})\,t}$.

12. Assertion If $x = \frac{a^n}{b^m}$ the $\frac{\Delta x}{x} = n\left(\frac{\pm \Delta a}{a}\right) - m\left(\frac{\pm \Delta b}{b}\right)$
The change in a or b *i.e.*, Δa or Δb may be comparable to a and b.
Reason The above relation is valid when $\Delta a << a$ and $\Delta b << b$.

13. Assertion Systematic errors and random errors fall in the same group of errors.
Reason Both systematic and random errors are based on the cause of error.

14. Assertion Absolute error may be negative or positive.
Reason Absolute error is the difference between the real value and the measured value of a physical quantity.

15. Assertion The watches having hour hand, minute hand and seconds hand have least count as 1 s.
Reason Least count is the maximum measurement that can be measured accurately by an instrument.

16. Assertion Pendulum bob is preferred to be spherical.
Reason Sphere has minimum surface area.

17. Assertion A screw gauge having a smaller value of pitch has greater accuracy.
Reason The least count of screw gauge is directly proportional to the number of divisions on circular scale.

Match the Columns

1. Match the following columns.

	Column I		Column II
(A)	R/L	(p)	Time
(B)	C/R	(q)	Frequency
(C)	E/B	(r)	Speed
(D)	$\sqrt{\varepsilon_0 \mu_0}$	(s)	None

2. Match the following columns.

	Column I		Column II
(A)	Stress	(p)	Pressure
(B)	Strain	(q)	Energy density
(C)	Modulus of elasticity	(r)	Angle
(D)	Torque	(s)	Energy

3. Suppose force (F), area (A) and time (T) are the fundamental units, then the match the following columns.

	Column I		Column II
(A)	Work	(p)	$[A^{1/2}T^{-1}]$
(B)	Moment of inertia	(q)	$[FA^{1/2}]$
(C)	Velocity	(r)	$[FA^{1/2}T^2]$

4. Match the following columns.

	Column I		Column II
(A)	Electrical resistance	(p)	$[M^{-1}L^{-2}T^4A^2]$
(B)	Capacitance	(q)	$[ML^2T^{-2}A^{-2}]$
(C)	Magnetic field	(r)	$[ML^2T^{-3}A^{-2}]$
(D)	Inductance	(s)	$[MT^{-2}A^{-1}]$

5. Match the following two columns.

	Column I		Column II
(A)	GM_eM_s	(p)	$[M^2L^2T^{-3}]$
(B)	$\frac{3RT}{M}$	(q)	$[ML^3T^{-2}]$
(C)	$\frac{F^2}{q^2B^2}$	(r)	$[L^2T^{-2}]$
(D)	$\frac{GM_e}{R_e}$	(s)	None

Entrance Corner

1. The SI unit of activity of a radioactive sample is **[J& K CET 2011]**
(a) Curie
(b) Rutherford
(c) Becquerel
(d) Millicurie

2. SI unit of power is **[J& K CET 2011]**
(a) Joule (b) Erg
(c) Newton (d) Watt

3. The SI unit of thermal conductivity is **[J& K CET 2011]**
(a) $Jsm^{-1}K^{-1}$
(b) $W^{-1}m^{-1}K^{-1}$
(c) $Wm^{-1}K^{-1}$
(d) $Wm^{-2}K^{-1}$

4. The dimensions of $(\mu_0\varepsilon_0)^{-1/2}$ are **[CBSE AIPMT 2011]**
(a) $[L^{-1}T]$ (b) $[LT^{-1}]$
(c) $[L^{-1/2}T^{1/2}]$ (d) $[L^{1/2}T^{-1/2}]$

5. Surface tension has the same dimensions as that of **[Kerala CEE 2011]**
(a) coefficient of viscosity
(b) impulse
(c) momentum
(d) spring constant
(e) frequency

6. The dimension of impulse is **[J&K CET 2011]**
(a) $[MLT^{-1}]$ (b) $[ML^2T^{-1}]$
(c) $[ML^{-1}T^{-1}]$ (d) $[MT^{-1}]$

7. If C be the capacitance and V be the electric potential, then the dimensional formula of CV^2 is **[KCET 2011]**
(a) $[ML^2T^{-2}A^0]$ (b) $[MLT^{-2}A^{-1}]$
(c) $[M^0LT^{-2}A^0]$ (d) $[ML^{-3}TA]$

8. What is the dimension of surface tension? **[WB JEE 2011]**
(a) $[MLT^0]$ (b) $[MLT^{-1}]$
(c) $[ML^0T^{-2}]$ (d) $[ML^0T^{-2}]$

9. The unit of magnetic moment is **[Guj. CET 2010]**
(a) TJ^{-1} (b) JT^{-1}
(c) Am^{-2} (d) Am^{-1}

10. Unit of electrical conductivity is **[UP CPMT 2010]**
(a) ohm (b) siemen
(c) m / mho (d) mho / m

11. From the dimensional consideration which of the following equations is correct? **[Haryana PMT 2010]**
(a) $T = 2\pi\sqrt{\frac{R^3}{GM}}$ (b) $T = 2\pi\sqrt{\frac{GM}{R^3}}$
(c) $T = 2\pi\sqrt{\frac{GM}{R^2}}$ (d) $T = 2\pi\sqrt{\frac{R^2}{GM}}$

12. If force F, length L and time T be considered fundamental units, then units of mass will be **[VMMC 2010]**
(a) $[FLT^{-2}]$ (b) $[FL^{-2}T^{-1}]$
(c) $[FL^{-1}T^2]$ (d) $[F^2LT^{-2}]$

13. Dimensions of capacitance is **[Manipal 2010]**
(a) $[M^{-1}L^{-2}T^4A^2]$ (b) $[MLT^{-3}A^{-1}]$
(c) $[ML^2T^{-3}A^{-1}]$ (d) $[M^{-1}L^{-2}T^3A^{-1}]$

14. A uniform wire of length L, diameter D and density ρ is stretched under a tension T. The correct relation between its fundamental frequency f, the length L and the diameter D is **[KCET 2010]**
(a) $f \propto \frac{1}{LD}$ (b) $f \propto \frac{1}{L\sqrt{D}}$
(c) $f \propto \frac{1}{D^2}$ (d) $f \propto \frac{1}{LD^2}$

15. The dimensions of resistance are same as those of ...... where h is the Planck's constant, e is the charge. **[KCET 2010]**
(a) $\frac{h^2}{e^2}$ (b) $\frac{h^2}{e}$
(c) $\frac{h}{e^2}$ (d) $\frac{h}{e}$

16. The equation of state of some gases can be expressed as $\left(p + \frac{a}{V^2}\right)(V - b) = RT$ where, p is absolute the pressure, V is the volume, T is absolute temperature and a and b are constants. The dimensional formula of a is **[JCECE 2010]**
(a) $[ML^5T^{-2}]$ (b) $[M^{-1}L^5T^{-2}]$
(c) $[ML^{-1}T^{-2}]$ (d) $[ML^{-5}T^{-2}]$

17. The relation $p = \frac{\alpha}{\beta} e^{\frac{-\alpha Z}{k\theta}}$, where p is pressure, Z is distance, k is Boltzmann constant and θ is temperature. The dimensional formula of β will be **[AFMC 2010]**
(a) $[M^0L^2T^0]$ (b) $[ML^2T]$
(c) $[ML^0T^{-1}]$ (d) $[M^0L^2T^{-1}]$

18. The dimension of electromotive force in terms of current A is **[BVP 2010]**
(a) $[ML^{-2}A^{-2}]$ (b) $[ML^2T^{-2}A^{-2}]$
(c) $[ML^2T^{-2}A^{-2}]$ (d) $[ML^2T^{-3}A^{-1}]$

19. The dimensional formula of $\frac{1}{\mu_0\varepsilon_0}$ is **[Guj. CET 2010]**
(a) $[M^0LT^{-2}]$ (b) $[M^0L^{-2}T^{-2}]$
(c) $[M^0LT^{-1}]$ (d) $[M^0L^2T^{-2}]$

20. If $p = \frac{RT}{V - b} e^{-\alpha V/RT}$, then dimensional formula of α is **[UP CPMT 2010]**
(a) p (b) R
(c) T (d) V

21. Velocity v is given by $v = at^2 + bt + c$, where t is time. What are the dimensions of a, b and c respectively? **[UP CPMT 2010]**
(a) $[LT^{-3}]$, $[LT^{-2}]$ and $[LT^{-1}]$ (b) $[LT^{-1}]$, $[LT^{-2}]$ and $[LT^{-3}]$
(c) $[LT^{-2}]$, $[LT^{-3}]$ and $[LT^{-1}]$ (d) $[LT^{-1}]$, $[LT^{-3}]$ and $[LT^{-2}]$

22. If E, M, L and G denote energy, mass, angular momentum and gravitation constant respectively, then the quantity (E^2L^2 / M^5G^2) has the dimensions of **[AMU 2010]**
(a) angle (b) length
(c) mass (d) None of these

23. A capillary tube is attached horizontally to a constant heat arrangement. If the radius of the capillary tube is increased by 10%, then the rate of flow of liquid will change nearly by **[WB JEE 2010]**
(a) + 10% (b) + 46%
(c) − 10% (d) − 40%

24. If momentum is increased by 20%, then kinetic energy increases by **[WB JEE 2010]**
(a) 48% (b) 44%
(c) 40% (d) 36%

25. If increase in linear momentum of a body is 50%, then change in its kinetic energy is **[Manipal 2010]**
(a) 25% (b) 125%
(c) 150% (d) 50%

26. At constant temperature, the volume of a gas is to be decreased by 4%. The pressure must be increased by **[BVP 2010]**
(a) 4% (b) 4.16%
(c) 8% (d) 3.86%

27. Choose the incorrect statement out of the following. **[AMU 2010]**
(a) Every measurement by any measuring instrument has some errors
(b)

Every calculated physical quantity that is based on measured values has some error
(c) A measurement can have more accuracy but less precision and *vice versa*
(d) The percentage error is different form relative error

28. Which one of the following quantities has not been expressed in proper units? **[Kerala CEE 2009]**

(a)	Torque	Newton metre
(b)	Stress	Newton metre^{-2}
(c)	Modulus of elasticity	Newton metre^{-2}
(d)	Power	Newton metre/second^{-1}
(e)	Surface tension	Newton metre^{-2}

29. The unit of specific conductivity is **[Manipal 2009]**
(a) Ω - cm^{-1} (b) Ω - cm^{-2}
(c) Ω^{-1} - cm (d) Ω^{-1} - cm^{-1}

30. An object is moving through the liquid. The viscous damping force action on it is proportional to the velocity. Then dimensional formula of constant of proportionality is **[UP CPMT, Punjab PMET 2009]**
(a) $[ML^{-1}T^{-1}]$ (b) $[MLT^{-1}]$
(c) $[M^0LT^{-1}]$ (d) $[ML^0T^{-1}]$

31. By what percentage should the pressure of a given mass of a gas be increased, so as to decrease its volume by 10% at a constant temperature? **[AIIMS 2009]**
(a) 5% (b) 7.2%
(c) 12.5% (d) 11.1%

32. Percentage error in the measurement of mass and speed are 2% and 3% respectively. The error in the estimation of kinetic energy obtained by measuring mass and speed will be **[AIIMS 2009]**
(a) 12% (b) 10%
(c) 2% (d) 8%

33. If the length of a seconds pendulum is increased by 2% then in a day the pendulum **[Kerala CEE 2009]**
(a) loses 764 s (b) loses 924 s
(c) gains 236 s (d) loses 864 s
(e) gains 346 s

Answers

Objective Problems (Level 1)

1. (d)	**2.** (b)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (b)	**9.** (b)	**10.** (c)
11. (c)	**12.** (d)	**13.** (a)	**14.** (c)	**15.** (d)	**16.** (d)	**17.** (b)	**18.** (a)	**19.** (d)	**20.** (b)
21. (c)	**22.** (c)	**23.** (c)	**24.** (a)	**25.** (b)	**26.** (a)	**27.** (d)	**28.** (c)	**29.** (a)	**30.** (d)
31. (d)	**32.** (d)	**33.** (d)	**34.** (c)	**35.** (b)	**36.** (d)	**37.** (b)	**38.** (c)	**39.** (c)	**40.** (a)
41. (c)	**42.** (a)	**43.** (d)	**44.** (c)	**45.** (c)	**46.** (b)	**47.** (b)	**48.** (b)	**49.** (a)	**50.** (a)
51. (a)	**52.** (b)	**53.** (d)	**54.** (c)	**55.** (b)	**56.** (b)	**57.** (a)	**58.** (b)	**59.** (b)	**60.** (c)
61. (b)	**62.** (a)	**63.** (a)	**64.** (d)	**65.** (a)	**66.** (d)	**67.** (d)	**68.** (b)	**69.** (c)	**70.** (a)
71. (a)	**72.** (c)	**73.** (b)	**74.** (d)	**75.** (a)	**76.** (d)	**77.** (b)	**78.** (c)	**79.** (c)	**80.** (c)
81. (b)	**82.** (a)	**83.** (c)	**84.** (d)	**85.** (d)	**86.** (c)	**87.** (b)	**88.** (d)	**89.** (b)	**90.** (c)

Objective Problems (Level 2)

1. (d)	**2.** (d)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (c)	**10.** (b)
11. (c)	**12.** (b)	**13.** (a)	**14.** (a)	**15.** (c)					

Assertion and Reason

1. (a)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (a)	**6.** (c)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (d)
11. (b)	**12.** (d)	**13.** (a)	**14.** (a)	**15.** (d)	**16.** (a)	**17.** (c)			

Match the Columns

1. ($A \to q$, $B \to p$, $C \to r$, $D \to s$)
2. ($A \to r$, $B \to p$, $C \to s$, $D \to q$)
3. ($A \to q$, $B \to r$, $C \to p$)
4. ($A \to s$, $B \to p$, $C \to r$, $D \to q$)
5. ($A \to q$, $B \to r$, $C \to r$, $D \to s$)

Entrance Corner

1. (c)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (d)	**6.** (a)	**7.** (a)	**8.** (c)	**9.** (b)	**10.** (d)
11. (a)	**12.** (c)	**13.** (a)	**14.** (a)	**15.** (c)	**16.** (a)	**17.** (a)	**18.** (d)	**19.** (d)	**20.** (a)
21. (a)	**22.** (d)	**23.** (b)	**24.** (b)	**25.** (b)	**26.** (b)	**27.** (d)	**28.** (c)	**29.** (d)	**30.** (c)
31. (d)	**32.** (d)	**33.** (d)							

Solutions

Objective Problems (Level 1)

1. Leap year, year and shake are the units of time.
3. 1 light year $= (3 \times 10^5)(365)(24)(3600)$
$= 9.416 \times 10^{12}$ km
8. Impulse = change in linear momentum.
13. Solid angle, strain and dielectric constant are dimensionless constant.
14. Since $(mvr) = n \cdot \frac{h}{2\pi}$
and $E = h\nu$
So, unit of h = joule second = angular momentum
17. Wb/m^2 and tesla are the units of magnetic field.
21. Impulse = Force × time
24. Young's modulus and pressure have the same dimensions.
26. Action is a force.
28. Relative density $= \frac{\text{Density of substance}}{\text{Density of water at } 4°\text{C temperature}}$
= Dimensionless
36. $m \propto v^a \rho^b g^c$. Writing the dimensions on both sides
$$[M] = [LT^{-1}]^a\ [ML^{-2}]^b\ [LT^{-2}]^c$$
$$[M] = [M^b L^{a-3b+c} T^{-a-2c}]$$
∴ $b = 1$
$a - 3b + c = 0$
$-a - 2c = 0$
Solving these we get
$a = 6$
Hence, $m \propto v^6$
37. Since $p^x Q^y c^z$ is dimensionless. Therefore,
$$[ML^{-1}T^{-2}]^x\ [MT^{-3}]^y\ [LT^{-1}]^z = [M^0L^0T^0]$$
Only option (b) satisfies this expression
So $x = 1, y = -1, z = 1$
38. Since units of length, velocity and force and doubled
Hence, $[m] = \frac{[\text{force}]\ [\text{time}]}{[\text{velocity}]}$, $[\text{time}] = \frac{[\text{length}]}{[\text{velocity}]}$
Hence unit of mass, and time remains same.
Momentum is doubled.
40. Since, $R = \frac{\rho l}{A}$, where ρ is specific resistance.
∴ $[\rho] = \left[\frac{RA}{l}\right]$, $R = \frac{V}{i}$, $V = \frac{W}{Q}$
$[\rho] = [ML^3T^{-1}Q^{-2}]$
41. $i = i_0\{1 - e^{-t/(L/R)}\}$
Where $\frac{L}{R}$ is time constant and its dimension is same as for time.
42. CR is time constant.
44. ωk is dimensionless.
45. $[a] = \left[\frac{F}{t}\right]$ and $[b] = \left[\frac{F}{t^2}\right]$
47. $\frac{1}{2}\varepsilon_0 E^2$ is energy density or energy per unit volume.
48. $p = \frac{a - t^2}{bx}$, where p-pressure, t-time
$[pbx] = [a] = [t^2]$
Hence, $[b] = \frac{[t^2]}{[px]}$
Dimensions of $\frac{a}{b} = [px] = [MT^{-2}]$
49. Velocity gradient is change in velocity per unit length.
50. Unit of emf e is volt.
51. $[a] = \left[\frac{v}{t}\right] : [b] = [vt] : [c] = [t]$
54. $F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1q_2}{r^2}$
⇒ $\varepsilon_0 = \frac{1}{4\pi} \times \frac{q_1q_2}{Fr^2}$
⇒ $\varepsilon_0 = \frac{(\text{coulomb})^2}{\text{newton-metre}^2}$
55. From definition of time constant $t = RC$, where R is resistance and C is capacitance.
∴ $R = \frac{t}{C}$
$= \frac{[T]}{[M^{-1}L^{-2}T^4A^2]}$
$R = [ML^2T^{-3}A^{-2}]$
56. $M = NIA$
57. Since, $R = \frac{\rho l}{A}$, where ρ is specific resistance
$[\rho] = \left[\frac{RA}{l}\right]$, $R = \frac{V}{i}$, $V = \frac{W}{Q}$
$[\rho] = [ML^3T^{-1}Q^{-2}]$
68. $R = 0.16$ mm
Hence, $A = \pi R^2$
$= \frac{22}{7} \times (0.16)^2$
$= 0.080384$
Since radius has two significant figure so answer also will have two significant figures.
∴ $A = 0.080$
73. Minimum number of significant figure should be 1.
75. Radius of ball = 5.2 cm
$V = \frac{4}{3}\pi R^3$

$$\frac{\Delta V}{V} = 3\left(\frac{\Delta R}{R}\right)$$

$$\left(\frac{\Delta V}{V}\right) \times 100 = 3\left(\frac{0.2}{5.2}\right) \times 100$$

$$= 11\%$$

78. Since error in measurement of momentum is + 100%

$\therefore$ $p_1 = p,\ p_2 = 2p$

$$K_1 = \frac{P^2}{2m},\ K_2 = \frac{(2p)^2}{2m}$$

$$\%\text{ in } K = \left(\frac{K_2 - K_1}{K_1}\right) \times 100$$

$$= \left(\frac{4-1}{1}\right) \times 100$$

$$= 300\%$$

81. $H = i^2\ R\ t$

$\therefore$ % error in H = 2 (% error in i) + (% error in R) + (% error in t)

82. $p = \frac{F}{A} = \frac{F}{L^2} = FL^{-2}$

% error in pressure = (% error in F) + 2 (error in L)

= (4%) + 2 (2%)

= 8%

89. $f = \frac{1}{2\pi\sqrt{LC}}$

or $\sqrt{LC} = \frac{1}{2\pi f} = \frac{T}{2\pi}$

Thus, $\sqrt{LC}$ has the dimensions of time.

Objective Problems (Level 2)

3. Volume of cylinder

$$V = \pi r^2 L,\ r = \left(\frac{D}{2}\right)$$

$$\therefore \left(\frac{\Delta V}{V}\right) \times 100 = 2\left(\frac{\Delta D}{D}\right) \times 100 + \left(\frac{\Delta L}{L}\right) \times 100$$

$$= 2\left(\frac{0.01}{4.0}\right) \times 100 + \left(\frac{0.1}{0.5}\right) \times 100$$

$$= 2.5\%$$

4. Since error is measured for 400 observations instead of 100 observations. So error will reduce by 1/4 factor.

Hence, $= \frac{x}{4}$

5. Dimension of (ohm) R

$$= \frac{h}{e^2};\ (e = \text{charge} = \text{current} \times \text{time})$$

$$= \frac{[Et]}{[it]^2}$$

$$= \frac{P}{i^2} = (R) \text{ as } P = \left(\frac{E}{t}\right)$$

8. $[\phi] = [BS] = [MT^{-2}A^{-1}]\ [L^2] = [ML^2T^{-2}A^{-1}]$

10. $g = \frac{GM}{R^2} : K = \frac{1}{2} I\omega^2 = \frac{L^2}{2I}$

Further, L will remain constant.

$\therefore$ $K \propto \frac{1}{I}$

or $K \propto \frac{1}{\frac{2}{5}MR^2}$

or $K \propto R^{-2}$

and $g \propto R^{-2}$

11. Energy $= \frac{1}{2} mv^2$

$$[m] = \frac{[E]}{[v^2]} = [Ev^{-2}]$$

12. [Froce] = $[MLT^2]$

$$\therefore\ 1\text{ N} = \left(\frac{1}{100}\right)\left(\frac{1}{1000}\right)(3600)^2$$

= 129.6 units.

13. $[FL^{-1}T^2] = [MLT^{-2}]\ [L^{-1}]\ [T^2] = [M]$

14. Here, $\frac{\pi}{\lambda}(ct - x)$ is dimensionless.

Hence, $\frac{ct}{\lambda}$ is also dimensionless and unit of ct is same as that of x.

Therefore, unit of λ is same as that of x. Also unit of y is same as that of A, which is also that unit of x.

15. m is mass per unit length.

Match the Columns

3. $[A] = [L^2]$

$\therefore$ $[L] = [A^{1/2}]$

$[T] = [T]$

$[F] = [MLT^{-2}]$

$\therefore$ $[M] = [FL^{-1}T^2] = [FA^{1/2}T^2]$

Now, $[W] = [FL] = [FA^{1/2}]$

$[I] = [ML^2] = [FA^{-1/2}T^2A] = [FA^{1/2}\ T^2]$

$[V] = [LT^{-1}] = [A^{1/2}T^{-1}]$

Entrance Corner

1. The SI unit of activity of a radioactive sample is Becqueral.
2. The SI unit of power is watt.
3. The SI unit of thermal conductivity is $Wm^{-1}\ K^{-1}$.
4. This expression for speed of light and the dimensions of speed of light are $[LT^{-1}]$.
5. Surface tension has the same dimension as that of spring constant.
6. The dimension of impulse = $[MLT^{-1}]$.
7. We know, Energy, $E = CV^2$

Dimensions of CV^2 = Dimensions of energy, E

$= [ML^2T^{-2}]$

8. We know, $T = \frac{F}{l}$

$$\text{Dimensions of } T = \frac{\text{Dimensions of } F}{\text{Dimension of } L}$$

$$T = \frac{[\text{MLT}^2]}{[\text{L}]}$$

$$= [\text{ML}^0\text{T}^{-2}]$$

9. Magnetic moment is the strength of magnet. Its SI unit is $\text{A} \times \text{m}^2$ or N-m/T or JT^{-1}.

10. Unit of electrical conductivity is mho / m or siemem / m.

11. Taking $T = 2\pi\sqrt{\frac{R^3}{GM}}$

Substituting the dimensions,

LHS, T = [T]

RHS, $$2\pi\left|\sqrt{\frac{R^3}{GM}}\right| = \sqrt{\frac{[\text{L}]^3}{[\text{M}^{-1}\text{L}^3\text{T}^{-2}]\,[\text{M}]}}$$

$$= \sqrt{[\text{T}]^2} = [\text{T}]$$

Thus, LHS = RHS for $T = 2\pi\sqrt{\frac{R^3}{GM}}$

12. Let $[\text{M}] \propto [\text{F}^a\text{L}^b\text{T}^c]$

So, using dimensions, we have

$$[\text{M}^1\text{L}^0\text{T}^0] = K[\text{MLT}^{-2}]^a\ [\text{L}]^b\ [\text{T}]^c$$

$\Rightarrow$ $a = 1, a = b = 0 \Rightarrow b = -1$

and $-2a + c = 0 \Rightarrow c = 2$

So unit of mass is $[\text{FL}^{-1}\text{T}^2]$

13. The capacitance C of a conductor is defined as the ratio of charge q given to raise the potential V of the conductor.

i.e., $C = \frac{q}{V}$

$$\therefore \text{Farad} = \frac{\text{coulomb}}{\text{volt}} = \frac{\text{coulomb}}{\text{joule / coulomb}}$$

$$= \frac{\text{coulomb}^2}{\text{joule}}$$

$$= \frac{(\text{ampera - sec})^2}{\text{newton - metre}} = \frac{\text{ampere}^2\text{ - sec}}{(\text{kg - m sec}^{-2}) \times \text{metre}}$$

$$= \frac{\text{ampera}^2\text{ - sec}^4}{\text{kg - metre}^2}$$

$$= \text{kg}^{-1}\text{- metre}^{-2}\text{ - sec}^4\text{ - amp}^2$$

So, the dimension of capacitance is $[\text{M}^{-1}\text{L}^{-2}\text{T}^4\text{A}^2]$.

14. The fundamental frequency is $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$$\Rightarrow \quad f = \frac{1}{2L}\sqrt{\frac{T}{\rho\pi\frac{D^2}{4}}} = \frac{1}{LD}\sqrt{\frac{T}{\pi\rho}}$$

$$\therefore \quad f \propto \frac{1}{LD}$$

15. Resistance, $R = \frac{V}{i} = \frac{W}{qi} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{A}^2\text{T}]}$

$$\Rightarrow \quad R = [\text{ML}^2\text{T}^{-3}\text{A}^{-2}]$$

Now for $$\left[\frac{h}{e^2}\right] = \frac{[\text{ML}^2\text{T}^{-1}]}{[\text{AT}]^2} = [\text{ML}^2\text{T}^{-3}\text{A}^{-2}]$$

16. In the equation p, V and T are pressure, volume and temperature respectively

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

Dimensions of $\frac{a}{V^2}$ will be same as that of pressure

$\therefore$ Dimensions of $\frac{a}{V^2}$ = dimensions of p

Dimension of a = dimension of p × dimension of V^2

$$[\text{ML}^{-1}\text{T}^{-2}]\ [\text{L}^6] = [\text{ML}^5\text{T}^{-2}]$$

17. In the given equation, $\frac{\alpha Z}{k\theta}$ should be dimensionless

$$\therefore \quad \alpha = \frac{k\theta}{Z}$$

$$\Rightarrow \quad [\alpha] = \frac{[\text{ML}^2\text{T}^{-2}\text{K}^{-1}]\ [\text{K}]}{[\text{L}]} = [\text{MLT}^{-2}]$$

and $$p = \frac{\alpha}{\beta}$$

$$\Rightarrow \quad [\beta] = \left[\frac{\alpha}{p}\right] = \frac{[\text{MLT}^{-2}]}{[\text{ML}^{-1}\text{T}^{-2}]}$$

$$= [\text{M}^0\text{L}^2\text{T}^0]$$

18. Electromotive force = potential difference

$$\Rightarrow \quad V = \frac{W}{q} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{AT}]}$$

$$= [\text{ML}^2\text{T}^2\text{A}^{-1}]$$

19. Velocity of electromagnetic waves $v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$\Rightarrow \quad \frac{1}{\mu_0\varepsilon_0} = v^2$$

Thus the dimensional formula of $\frac{1}{\mu_0\varepsilon_0}$

$$= [\text{M}^0\text{LT}^{-1}]^2$$

$$= [\text{M}^0\text{L}^2\text{T}^{-2}]$$

20. Given $p = \frac{RT}{V - b}\,e^{-\alpha V/RT}$

So, $\frac{\alpha V}{RT}$ is dimensionless.

Hence, $$[\alpha] = \left[\frac{RT}{V}\right] = \frac{[\text{ML}^2\text{T}^{-2}\theta^{-1}]\ [\theta]}{[\text{L}^3]}$$

$$= [\text{ML}^{-1}\text{T}^{-2}]$$

This is also the dimension less formula of pressure.

21. Dimensions of velocity is $[\text{v}] = [\text{L}]\ [\text{T}^{-1}]$

So, dimensions of $[\text{at}^2] = [\text{LT}^{-1}]$

$$\Rightarrow \quad [\text{a}]\ [\text{T}^2] = [\text{LT}^{-1}]$$

$$\Rightarrow \quad [\text{a}] = [\text{LT}^{-3}]$$

Dimensions of $[\text{bt}] = [\text{LT}^{-1}] \Rightarrow [\text{b}]\ [\text{T}] = [\text{LT}^{-1}]$

$$\Rightarrow \quad [\text{b}] = [\text{LT}^{-2}]$$

Dimensions of $[\text{c}] = [\text{LT}^{-1}]$

22. The dimension of $E = [ML^2T^{-2}]$

Dimensions of $M = [M]$

Dimensions of $L = [ML^2T^{-1}]$

Dimensions of $G = [M^{-1}L^3T^{-2}]$

$\therefore$ Dimensions of

$$\left[\frac{E^2L^2}{M^5G^2}\right] = \frac{[ML^2T^{-2}]^2\ [ML^2T^{-1}]^2}{[M]^5\ [M^{-1}L^3T^{-2}]^2} = [ML^2T^{-2}]$$

23. Volume of liquid coming out of the tube per second

$$\Rightarrow \quad V = \frac{p\pi r^4}{8\eta l}$$

$$\Rightarrow \quad \frac{V_2}{V_1} = \left[\frac{r_2}{r_1}\right]^4$$

$$\Rightarrow \quad V_2 = V_1\left[\frac{110}{100}\right]^4$$

$$= V_1\,(1.1)^4 = 1.4641 \text{ volt}$$

$$\Rightarrow \quad \frac{\Delta V}{V} = \frac{V_2 - V_1}{V} = \frac{1.4641 - V}{V} = 46\%$$

24. The kinetic energy is given by $KE = \dfrac{p^2}{2m}$

So, $$\Delta KE = \frac{2p\Delta p}{2m} = \frac{p\Delta p}{m}$$

$$\Rightarrow \quad \frac{4KE}{KE} = \frac{2\Delta p}{p}$$

Thus, the final momentum becomes 1.2 p.

So, percentage change in $KE = \dfrac{\text{final KE} - \text{initial KE}}{\text{initial KE}} \times 100$

$$= \frac{1.44\,(p^2/2m) - (p^2/2m)}{(p^2/2m)} = 44\%$$

25. We know that linear momentum $p = \sqrt{2mK}$

Now we have $p_1 = p$, $p_2 = p_1 + 50\%$ of $p_1 = 1.5p_1$

$$\Rightarrow \quad \frac{K_1}{K_2} = \frac{p_1^2}{p_2^2} \Rightarrow K_2 = \frac{p_2^2}{p_1^2}\,K_1 = 2.25\ K$$

So change in $KE = 2.25 - 1 = 1.25 = 125\%$

26. At constant temperature, $p_1V_1 = p_2V_2$

$$\frac{p_1}{p_2} = \frac{V_2}{V_1}$$

Hence, tractional change in volume

$$\Rightarrow \quad \frac{V_1 - V_2}{V_1} = \frac{4}{100} = \frac{1}{25}$$

$$\Rightarrow \quad 1 - \frac{V_2}{V_1} = \frac{1}{25} \Rightarrow \frac{V_2}{V_1} = \frac{24}{25}$$

$$\Rightarrow \quad \frac{p_1}{p_2} = \frac{V_2}{V_1} = \frac{24}{25}$$

$$\Rightarrow \quad \frac{p_2 - p_1}{p_1} = \frac{25}{24} - 1 = \frac{1}{24}$$

Percentage increase in pressure $= \dfrac{100}{24} = 4.16\%$

27. When the relative error is expressed in percentage, we cal it percentage error.

28. The correct unit of surface tension is newton/metre.

29. Specific conductivity $= \dfrac{1}{\text{specific resistance}} = \dfrac{1}{\Omega\text{-cm}}$

$$= (\Omega\text{-cm})^{-1}$$

30. We have $F \propto v \Rightarrow F = kv$

$$\Rightarrow \quad [k] = \left[\frac{F}{v}\right] = \left[\frac{[MLT^{-2}]}{[LT^{-1}]}\right] = [ML^0T^{-1}]$$

31. When T is constant, $pV = \text{constant}$. When volume is decreased by 10% that is volume becomes $\dfrac{90}{100}$, the pressure must become 100/99. Thus percentage increase in pressure

$$= \frac{(100 - 90) \times 100}{90} = 11.1\%$$

32. Kinetic energy $K = \dfrac{1}{2}mv^2$

Fractional error in kinetic energy

$$\frac{\Delta K}{K} = \frac{\Delta m}{m} + \frac{2\Delta v}{v}$$

Percentage error in kinetic energy is

$$= \frac{\Delta m}{m} \times 100 + \frac{2\Delta v}{v} \times 100$$

As we know, $\dfrac{\Delta m}{m} \times 100 = 2\%$ and $\dfrac{2\Delta v}{v} \times 100 = 3\%$

So, percentage error in kinetic energy

$$= 2 + 2 \times 3 = 2 + 6 = 8\%$$

33. Time period, $T = 2\pi\sqrt{\dfrac{l}{g}}$ or $\dfrac{\Delta T}{T} = \dfrac{1}{2}\dfrac{\Delta l}{l}$

For 1s, $\Delta T = \dfrac{1}{2}\left(\dfrac{\Delta l}{l}\right)T = \dfrac{1}{2} \times 0.02 \times T = 0.01\,T = 0.01$ s

For a day, $\Delta T = 24 \times 60 \times 60 \times 0.01 = 864$ s